AD-784 375

AN AZIMUTH DETERMINATION SYSTEM USING THE NAVY NAVIGATION SATELLITES

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Prepared for:

Naval Plant Representative Office
Advanced Research Projects Agency

May 1974

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1. REPORT NUMBER TG 1235	2. GOVT ACCESSION NO	3. RECIPIENT'S CATALOG NUMBER AD 784 375
4. TITLE (and Subtitle) An Azimuth Determination System Using the Navy Navigation Satellites		5. TYPE OF REPORT & PERIOD COVERED Technical Memorandum
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J. R. Albertine		8. CONTRACT OR GRANT NUMBER(s) N00017-72-C-4401
9. PERFORMING ORGANIZATION NAME & ADDRESS The Johns Hopkins University Applied Physics Laboratory 8621 Georgia Avenue Silver Spring, Maryland 20910		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AO 1614
11. CONTROLLING OFFICE NAME & ADDRESS Navy Plant Representative Office 8621 Georgia Ave. Silver Spring, Md. 20910		12. REPORT DATE May 1974
		13. NUMBER OF PAGES 84
14. MONITORING AGENCY NAME & ADDRESS Navy Plant Representative Office 8621 Georgia Ave. Silver Spring, Maryland 20910		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Navigation
Satellite Navigation
Navy Navigation Satellite System
Azimuth Determination

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A system has been developed using the Navy Navigation Satellites that can provide azimuth information on a worldwide basis. This capability is particularly useful in the polar regions where conventional navigation systems based on gyrocompasses experience alignment difficulties. In this newly developed system, two antennas separated by 50 to 100 meters are used as an interferometer array to measure phase differences of a satellite signal. Based on this information, it is possible to determine the azimuth of the baseline connecting the two receiving antennas.

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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

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Technical Memorandum

AN AZIMUTH DETERMINATION SYSTEM USING THE NAVY NAVIGATION SATELLITES

J. R. ALBERTINE

SPONSORED BY ARPA UNDER AO 1614

THE JOHNS HOPKINS UNIVERSITY • APPLIED PHYSICS LABORATORY
8621 Georgia Avenue ◦ Silver Spring, Maryland ◦ 20910
Operating under Contract N00017-72-C-4401 with the Department of the Navy

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A system has been developed using the Navy Navigation Satellites to provide azimuth information on a worldwide basis. This capability is particularly useful in the polar regions where conventional navigation systems based on gyrocompasses are handicapped by gyro alignment difficulties. In this newly developed system, two antennas separated by 50 to 100 meters are used as an interferometer array to measure the phase difference of a satellite signal. Based on this information, it is possible to determine the azimuth of the baseline connecting the two receiving antennas.

FOREWORD

Precision navigation has been a problem ever since man began to travel into unfamiliar areas. Today the problem is crucial whether it is related to driving a car on an unfamiliar road or piloting an airplane across an ocean, where no markers exist.

The goal of navigation can be simply stated as successful travel from an arbitrary location to a desired location in an optimal manner. To solve this problem the navigator needs two pieces of information: his current location and a reference direction. (It is assumed that the navigator knows the location of his destination and all intervening disturbances.) To be useful, this information should be readily available over the total area of possible travel.

The reference direction can be defined in terms of azimuth. Azimuth is an angle referenced to the earth's spin axis such that an angle of 0° is called north, east is 90°, south is 180°, and west is 270°.

A solution to the problem of determining a reference direction will be treated here. The proposed technique uses the radio signal received from a satellite whose location is known as a function of time and two nondirectional antennas used as an interferometer. This system has been demonstrated using the Navy navigation satellites.

The interferometer method offers many advantages over other "north seeking" techniques. The most common of these methods are starsighting and north-seeking gyro compasses. Starsighting systems are weather limited and primarily useful at night. North-seeking gyro compasses, on the other hand, overcome the weather and night limitations but will not self-align in the high latitude polar regions and are subject to drift errors. The interferometer azimuth determination system overcomes the disadvantages of both of these methods by allowing measurements on a worldwide basis, independent of weather or time of day.

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1. INTRODUCTION

THE NAVY NAVIGATION SATELLITE SYSTEM

The Navy Navigation Satellite System (NAVSAT) developed by The Applied Physics Laboratory for the Department of the Navy consists of four or more near-earth satellites in polar orbits, tracking stations, injection stations, a computing center, and receiving navigation equipment. This system employs doppler tracking for both satellite position determination and navigation. To determine satellite position, four tracking stations in precisely known locations observe the doppler shift of the highly stable radio signals generated by the satellite transmitter as the satellite passes over the station. The doppler information allows the computing center to determine the satellite's position as a function of time. From this measured positional information, the location of the satellite at a future time can be predicted. These predictions are stored in the memory of the satellite by the injection station. As the satellite orbits the earth, it continually transmits its position and precise time. By receiving time and position data from the satellite and observing the doppler shift in the satellite signals a navigator can determine his position.

The Navy navigation satellites are in nominal 600 nmi circular polar orbits with periods of approximately 105 minutes. There are currently five such satellites with approximately equal angular separations between the orbital planes (Fig. 1) so that the coverage is fairly evenly spread, providing reception possibilities approximately once every 2 hours at the equator and considerably more frequently as latitude increases. Typically the satellite is in view for approximately 15 minutes during each pass.

The ground support system consists of tracking stations that receive, record, and digitize doppler signals from the satellites; a computing center that computes future orbits, orbital parameters, and time corrections; and an injection station that transmits these new orbital parameters and time

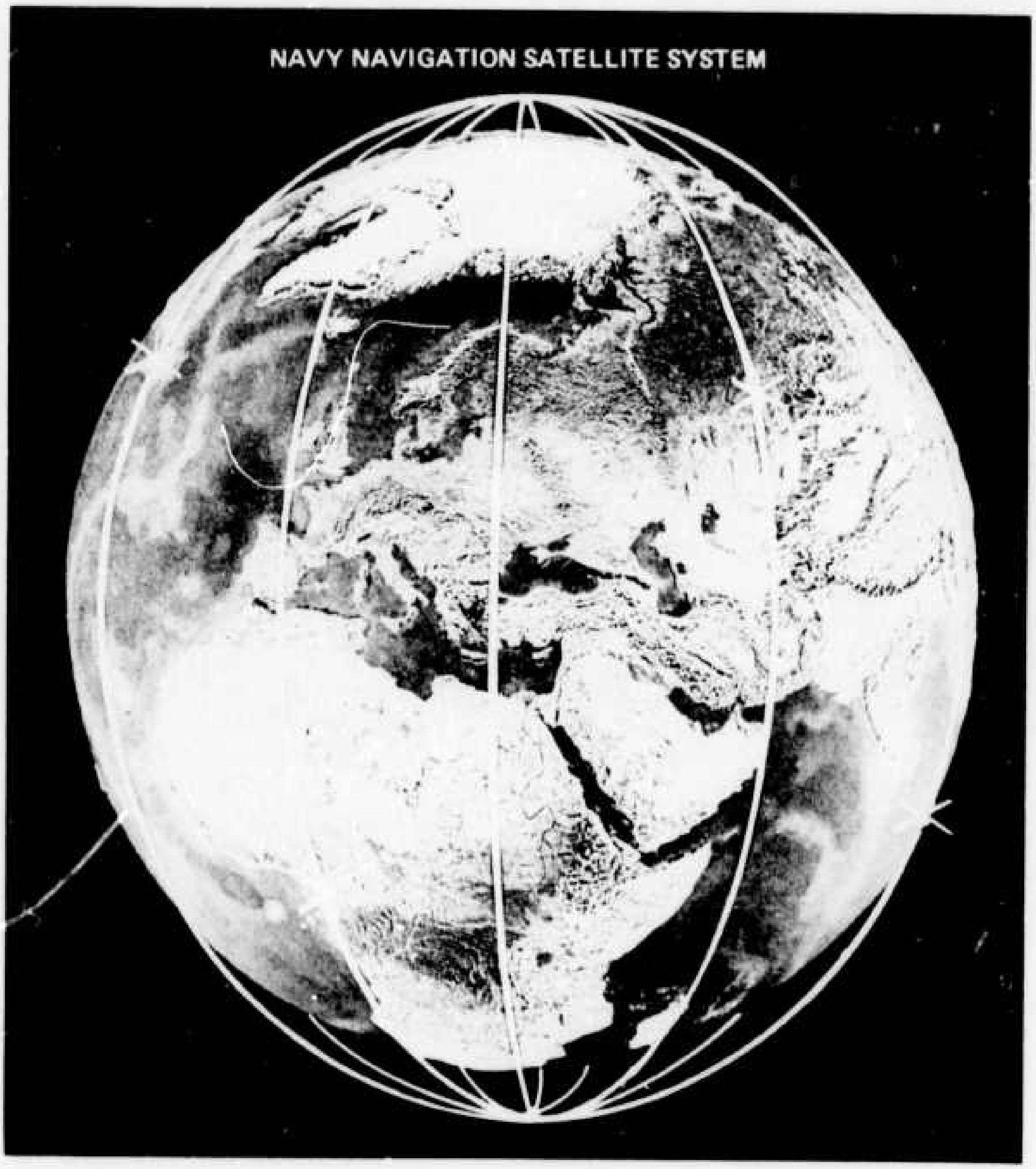

Fig. 1 THE NAVY NAVIGATION SATELLITES IN POLAR ORBIT

corrections to the satellite. In addition, the U.S. Naval Observatory recovers the satellite time signals and compares them with universal time. This information is forwarded to the computing center for the time correction computations.

Navigation satellite data can be used to aid navigation systems in a number of ways, such as providing survey bench marks, or to update the inertial platforms of ships or aircraft. These navigation tasks can be performed on a worldwide basis regardless of weather.

THE AZTRAN SYSTEM

Aztran, an abbreviation for "azimuth from Transit," is a system that determines north reference from a single pass of a navigation satellite. The system uses two antennas separated by a baseline distance of 30 to 200 meters. Because of the separation of the antennas, there is a transmission path length difference from the antennas to the satellite. From Fig. 2 it can be seen that the path difference may be computed from the equation:

$$\text{Path difference} = D \cdot \cos(E_L) \cdot \cos(Az_{sat} - Az_{ant}).$$

When the signals from the two antennas are compared, there is a phase difference caused by this path length difference:

$$\text{Phase difference} = (2\pi D/\lambda) \cdot \cos(E_L) \cdot \cos(Az_{sat} - Az_{ant}),$$

where λ is the wavelength of the received signal.

As the satellite moves across the sky, there are changes in the path length difference and the corresponding phase angle. This phase change is measured between time A and time B and is equal to:

$$\Delta\phi = \frac{2\pi D}{\lambda} \{[\cos(E_L)\cos(Az_{sat} - Az_{ant})]_B - [\cos(E_L)\cos(Az_{sat} - Az_{ant})]_A\}$$

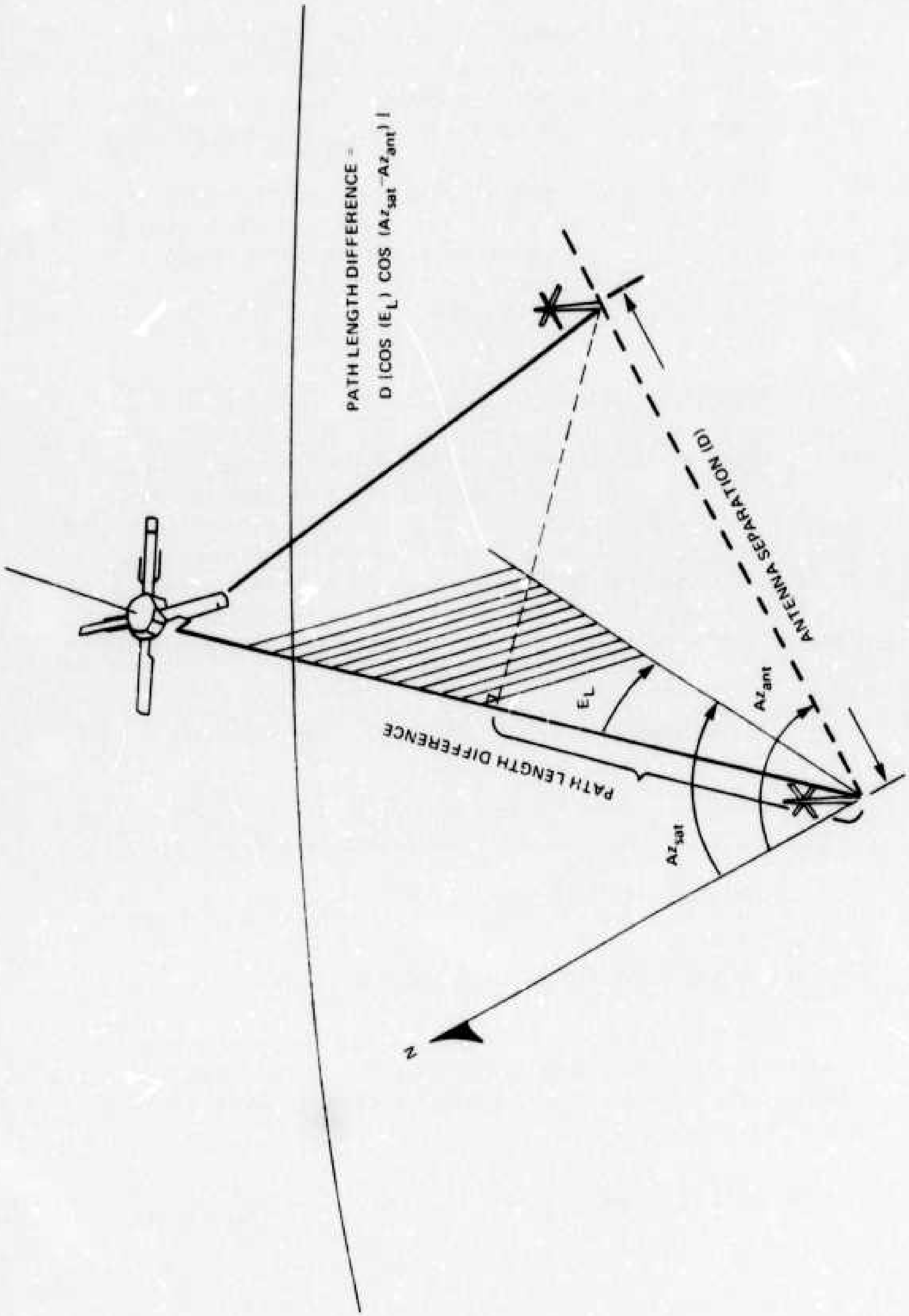

Fig. 2 AZTRAN GEOMETRY

In this equation (since the observer's and satellite's positions are known) the only unknowns are D, the antenna separation, and the antenna baseline azimuth (Az_{ant}). By making repeated measurements over fixed time intervals during a satellite pass, the baseline azimuth and antenna separation are determined.

APPLICATIONS

Possible applications of such a system are numerous. Accurate azimuth information, easily obtained in the field, could be used for gun fire control and for mobile missile field sites. This capability would free the missile crews from their dependence on presurveyed sites for this information and guarantee its availability day or night, regardless of weather. Another possible application is in the alignment of inertial platforms. Aztran would be particularly useful in the polar regions where an inertial platform will not self-align and must be referenced to an external azimuth source. Ships at sea and aircraft are other potential users.

2. THEORY OF OPERATION

ORBIT CALCULATIONS

The Navy navigation satellites transmit a message containing the predicted Keplerian elements of the satellite's orbit in inertial space, and a precise time mark every even 2 minutes. These Keplerian elements are then converted into Cartesian earth-centered coordinates, which rotate with the earth such that a fixed point on the earth has fixed x-y-z coordinates.

Consider the ellipse (PSA) shown in Fig. 3 with a circle (PCA) circumscribing the ellipse. Point O' is the center of the circle and the ellipse. Point O is at one focus of the ellipse which is the origin of the u-v coordinate system and the geocenter. The distance A_o is called the semi-major and B_o is called the semi-minor axis. Also:

$$B_o = A_o\sqrt{1 - \epsilon^2} ,$$

where ϵ is called the eccentricity of the ellipse and can vary from 0 (for a circle) to 1 (for a straight line).

If this ellipse represents the orbit in the u-v plane then O is the center of the earth and S is the position of the satellite at some arbitrary time t. The time at which the satellite is at its lowest point or perigee (point P) is called t_p.

The angle E as shown in Fig. 3 is called the accentric anomaly. The mean (average) motion of the satellite is defined as:

$$n = \frac{2\pi}{T} .$$

where T is the orbital period of the satellite. The mean angular motion (M) is the average angular motion in radians since the last time of perigee:

$$M = \frac{2\pi}{T} \cdot (t - t_p) = n \cdot (t - t_p)$$

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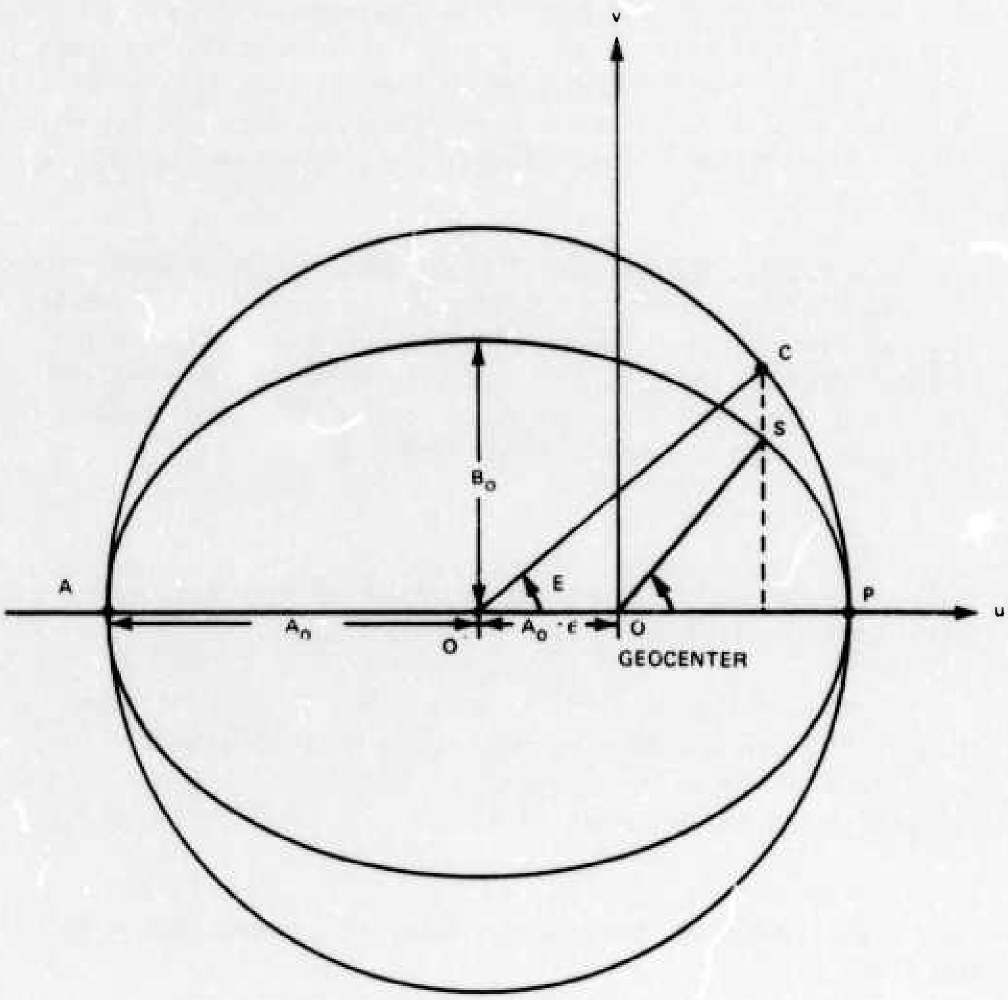

Fig. 3 ELLIPTIC ORBIT IN THE u-v PLANE

Therefore, given A_o, ϵ, n, and t_p, the u-v coordinates of the satellite at S can be found as a function of time as follows:

$$M(t) = n \cdot (t - t_p)$$

$$E(t) = M(t) + \epsilon \sin M(t) .$$

Then:

$$\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} A_o \cdot [\cos E(t) - \epsilon] \\ A_o \cdot \sqrt{1 - \epsilon^2} \sin E(t) \end{bmatrix} . \qquad (1)$$

Equation (1) would hold true for an unperturbed orbit about a spherical mass. The orbit of an earth satellite does not remain in an unperturbed orbit due to variations in the gravitational field, air drag on the satellite, and the effects of other solar bodies (primarily the sun and moon). Therefore, some of the orbital parameters which were treated as constants must be made functions of time, and cross-plane terms must be added.

Consider making the u-v plane into a right-hand Cartesian coordinate system by the addition of a w axis. Out-of-plane variations are then solely in this axis.

In the Navy navigation satellite system the semi-major axis and the eccentric anomaly are treated as functions of time. A cross-plane term, $\eta(t)$, is also introduced. The coordinates of the satellite in the u-v-w frame can now be found as follows, given A_o, $\Delta A(t)$, ϵ, n, t_p, $\Delta E(t)$, and $\eta(t)$:

$$M(t) = n(t - t_p)$$

$$E(t) = M(t) + \epsilon \sin M(t) + \Delta E(t)$$

$$A(t) = A_o + \Delta A(t)$$

$$\begin{bmatrix} u(t) \\ v(t) \\ w(t) \end{bmatrix} = \begin{bmatrix} A(t) \cdot [\cos E(t) - \epsilon] \\ A(t) \cdot \sqrt{1 - \epsilon^2} \sin E(t) \\ \eta(t) \end{bmatrix} . \tag{2}$$

The u-v-w coordinate system can now be transformed into an earth-centered nonrotating coordinate system. Define the X'-Y'-Z' coordinate system as having the origin at the center of the earth and the positive Z axis pointing north along the earth's spin axis. The X'-Y' plane lies in the equatorial plane and the positive X' axis points toward Aires (γ). This coordinate frame is fixed in inertial space.

Figure 4 shows the u-v-w and the X'-Y'-Z' coordinates. The origins of the two systems coincide and the v axis lies in the X'-Y'plane. The longitude of the ascending node (Ω_0) is the angle measured in the equatorial plane from the positive X' axis to the negative v axis. The orbital inclination (i) is the angle measured from the positive Z' axis to the positive w axis. The argument of perigee (ω) is the angle from the ascending node to the point of orbital perigee.

Given the preceding parameters defining the satellite's orbit, Eq. (3) is the transformation from the u-v-w coordinate system to the X'-Y'-Z' system:

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos(\Omega) & -\sin(\Omega) & 0 \\ \sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & -\sin(i) \\ 0 & \sin(i) & \cos(i) \end{bmatrix} \cdot \begin{bmatrix} \cos(\omega) & -\sin(\omega) & 0 \\ \sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{3}$$

Figure 5 shows the relationship between the X'-Y'-Z' coordinate system and the desired X-Y-Z frame which is fixed with respect to the earth's surface. The origins and Z axis of the two systems are coincident. ω_e is the earth's angular rotation rate. The X-Y plane and the X'-Y' plane are both equatorial and rotated by Λ_g, the Greenwich hour angle. If the angle β is defined as $\Lambda_g - \omega_e (t - t_p)$, then:

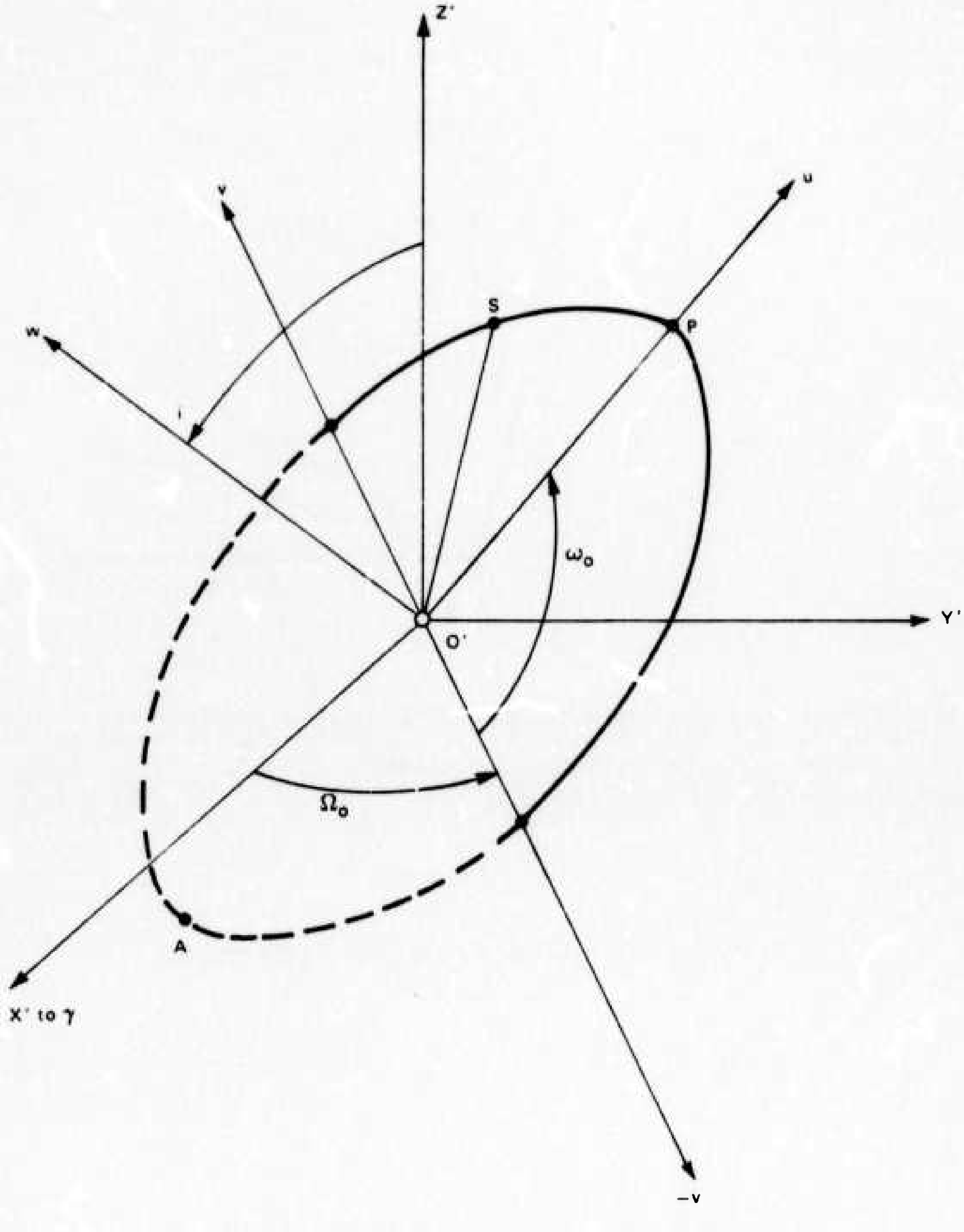

Fig. 4 THE u-v-w and X'-Y'-Z' COORDINATE FRAMES

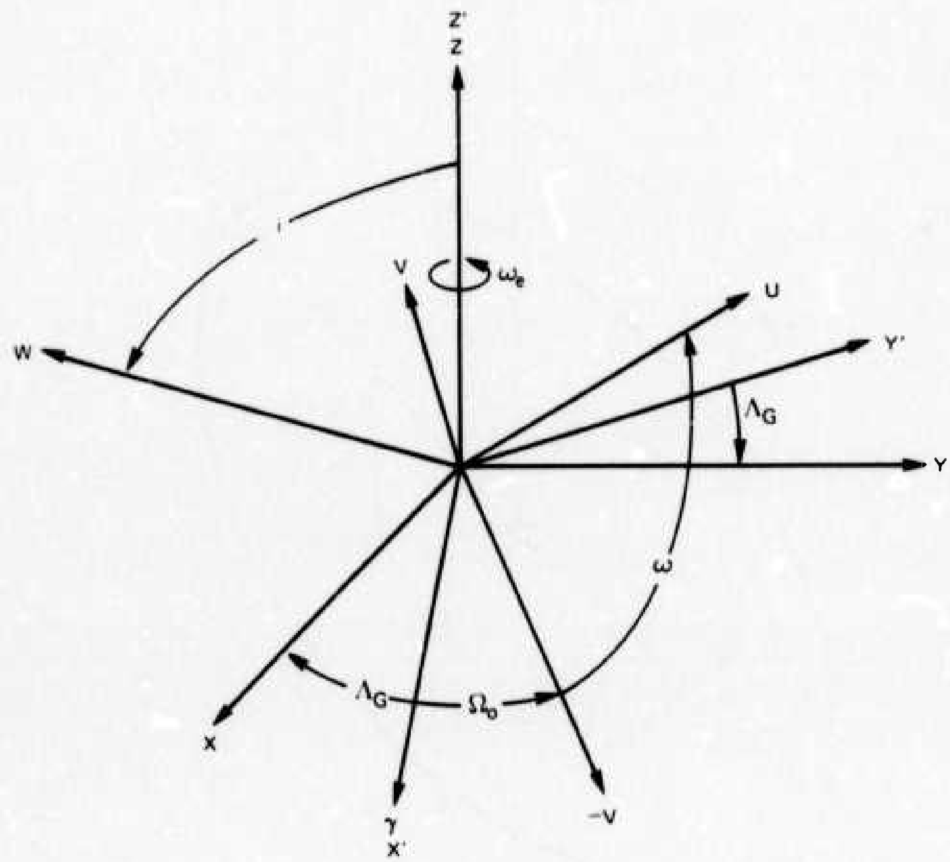

Fig. 5 THE X'-Y'-Z' AND X-Y-Z COORDINATE FRAMES

$$\begin{bmatrix} X' \\ Y' \\ Z \end{bmatrix} = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ -\sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} . \tag{4}$$

SATELLITE MESSAGE

During each 2 minute interval a Navy navigation satellite transmits 19 message words used for navigation. The first 8 are called ephemeral words and contain variables for the 3 past 2 minute intervals, the current interval, and the next 4 intervals. The variables contained in each ephemeral are:

t_k - the time of the interval,

ΔE_k - the variation in the eccentric anomaly,

ΔA_k - the variation in the semi-major axis, and

η_k - the out of plane distance.

The next 11 words received do not change with each 2 minute interval. They are:

t_p - time of satellite perigee,

n - mean motion of the satellite,

ω - the argument of perigee at t_p,

$\dot{\omega}$ - precession rate of perigee,

ϵ - orbital eccentricity,

A_o - semi-major axis,

Ω_o - right ascension of ascending node at t_p,

$\dot{\Omega}$ - precession rate of node,

Ci - cosine of the inclination,

Λ_g - longitude of Greenwich at t_p, and

Si - sine of the inclination.

Figure 6 is an example of a sequence of 2 minute messages received from the satellite. A minimum of three intervals is required to solve the navigation equations for the latitude and longitude of the observer and the difference in the frequencies of the satellite and receiver oscillators. This last quantity is not predictable and must be a variable of the solution. A satellite pass of maximum duration will be observable for seven or eight 2-minute intervals.

The set of orbital elements is then arranged as follows, where k represents a 2 minute interval during the pass, and T_k is the correct time at a 2 minute time mark:

$$t_k = T_k - t_p$$

$$M_k = n \cdot t_k$$

$$E_k = M_k + \epsilon \sin M_k + \Delta E_k$$

$$A_k = A_o + \Delta A_k$$

$$\begin{bmatrix} u_k \\ v_k \\ w_k \end{bmatrix} = \begin{bmatrix} A_k \cdot (\cos E_k - \epsilon) \\ A_k \cdot \sqrt{1 - \epsilon^2} \sin E_k \\ \eta_k \end{bmatrix} \qquad (5)$$

Equation (5) is then transformed into the X-Y-Z frame using Eq. (3) and (4) for each interval where:

$$\omega_k = \omega_o - \dot{\omega} \cdot t_k$$

$$\Omega_k = \Omega_o + \dot{\Omega} \cdot t_k$$

$$\beta_k = \Lambda_g - \omega_e \cdot t_k \ .$$

```
                         THE APL AZTRAN SYSTEM

420050084  630070213  640170404  200250662  210300960  220321289
230311620  240261955  029871340  836613670  835694710  800199950
800015910  807463480  834318690  900002660  800062340  809826660
809999810
430010259  640100470  200190757  210261070  220301413  230311775
240292110  250252425  094036060  836612930  835553830  800199470
800015640  807463480  834316980  900002380  800062390  825911760
809999810
640100470  200190757  210261070  220301413  230311775  240292110
250252425  260182684  094036060  836612930  835553830  800199470
800015640  807463480  834316980  900002380  800062390  825911760
809999810
200190757  210261070  220301413  230311775  240292110  250252425
260182684  270092895  094036060  836612930  835553830  800199470
800015640  807463480  834316980  900002380  800062390  825911760
809999810
210261070  220301413  230311775  240292110  250252425  260182684
270092895  080023026  094036060  836612930  835553830  800199470
800015640  807463480  834316980  900002380  800062390  825911760
809999810
220301413  230311775  240292110  250252425  260182684  270092895
080023026  090133075  094036060  836612930  835553830  800199470
800015640  807463480  834316980  900002380  800062390  825911760
809999810
230311775  240292110  250252425  260182684  270092895  080023026
090133075  400243049  094036060  836612930  835553830  800199470
800015640  807463480  834316980  900002380  800062390  825911760
809999810


VOTED MESSAGE

200190757  210261070  220301413  230311775  240292110  250252425
260182684  270092895  080023026  094036060  836612930  835553830
800199470  800015640  807463480  834316980  900002380  800062390
825911760  809999810


000243 777433 307700 000000 055300 541002 213700 557250 611440
000253 630523 654600 751474 740100 051350 117600 102243 105100
000263 112623 250400 777777 777740 777777 777740 777777 777740
000273 777777 777740 777777 777740 777777 777740

DAY.....  288
TIME....  1430
```

Fig. 6   EXAMPLE OF SEQUENCE OF 2 MINUTE SATELLITE MESSAGES

More information concerning the details of the solution of the navigation equations can be found in Technical Memorandum TG 819-1, Program Requirements for Two-Minute Integrated Doppler Satellite Navigation Solution edited by J. B. Moffett.

AZIMUTH EQUATIONS

In this section the equations are developed which allow the determination of the azimuth of a line connecting two antennas. The relative positions of the two antennas can be interpreted in terms of azimuth angle or in earth-fixed coordinates.

Recall the earth fixed X-Y-Z coordinate system from the previous section where the center of the earth was the origin, the equator lay in the X-Y plane and the Greenwich meridian lay in the X-Z plane (Fig. 7). If φ is the geodetic latitude and λ is the geodetic longitude, then:

$$X_n = \left\{ \frac{Re}{[\cos^2\varphi + (1-f)^2 \sin^2\varphi]^{\frac{1}{2}}} + Ht \right\} \cos\varphi \cos\lambda \tag{6}$$

$$Y_n = \left\{ \frac{Re}{[\cos^2\varphi + (1-f)^2 \sin^2\varphi]^{\frac{1}{2}}} + Ht \right\} \cos\varphi \sin\lambda \tag{7}$$

$$Z_n = \left\{ \frac{Re(1-f)^2}{[\cos^2\varphi + (1-f)^2 \sin^2\varphi]^{\frac{1}{2}}} + Ht \right\} \sin\varphi \quad , \tag{8}$$

where (X_n, Y_n, Z_n) are the coordinates of the navigator on the earth's surface whose latitude and longitude are respectively φ and λ, Re is the equatorial radius of the earth, Ht is the observer's height above the reference ellipsoid, and f is the coefficient of flattening of the reference ellipsoid.

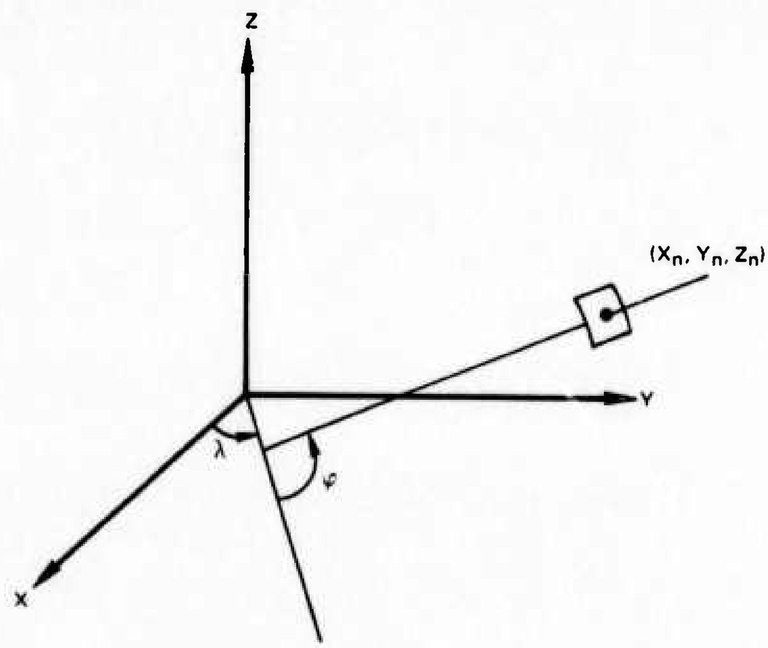

Fig. 7 EARTH-CENTERED CARTESIAN COORDINATE SYSTEM

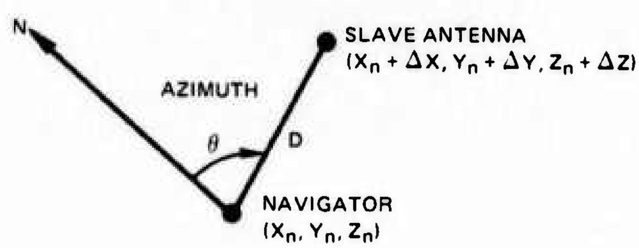

Fig. 8 RELATIVE LOCATION OF THE TWO ANTENNAS

Assume that the position of a satellite is known in the earth fixed coordinate frame and is (X_s, Y_s, Z_s). Then the slant range from satellite to navigator can be expressed as:

$$SR = [(X_s - X_n)^2 + (Y_s - Y_n)^2 + (Z_s - Z_n)^2]^{\frac{1}{2}}$$

$$= (X_{net}^2 + Y_{net}^2 + Z_{net}^2)^{\frac{1}{2}} , \qquad (9)$$

where

$$X_{net} = (X_s - Y_n), \text{ etc.}$$

Let the second antenna be located at ($X_n + \Delta X$, $Y_n + \Delta Y$, $Z_n + \Delta Z$) (Fig. 8), at an azimuth angle θ from North, and with an antenna separation D. Then:

$$D = (\Delta X^2 + \Delta Y^2 + \Delta Z^2)^{\frac{1}{2}} \quad . \qquad (10)$$

The difference in slant range from the satellite to the two antennas (ΔSR) can then be calculated:

$$\Delta SR = \left\{ [X_s - (X_n + \Delta X)]^2 + [Y_s - (Y_n + \Delta Y)]^2 + [Z_s - (Z_n + \Delta Z)]^2 \right\}^{\frac{1}{2}} - SR$$

$$= \left\{ [X_{net} - \Delta X]^2 + [Y_{net} - \Delta Y]^2 + [Z_{net} - \Delta Z]^2 \right\}^{\frac{1}{2}} - SR \quad . \qquad (11)$$

Expanding this results in:

$$\Delta SR = [SR^2 - 2(X_{net} \Delta X + Y_{net} \Delta Y + Z_{net} \Delta Z) + D^2]^{\frac{1}{2}} - SR \quad .$$

Neglecting (D^2) gives:

$$\Delta SR = SR \left[1 - \frac{2}{SR^2} (X_{net} \Delta X + Y_{net} \Delta Y + Z_{net} \Delta Z) \right]^{\frac{1}{2}} - SR \quad .$$

Taking the first term of the binominal expansion gives the following:

$$\Delta SR = SR - \frac{1}{SR}(X_{net}\,\Delta X + Y_{net}\,\Delta Y + Z_{net}\,\Delta Z) - SR$$

$$\Delta SR = -\frac{1}{SR}(X_{net}\,\Delta X + Y_{net}\,\Delta Y + Z_{net}\,\Delta Z)\ . \qquad (12)$$

The only remaining problem is to determine ΔX, ΔY, and ΔZ in terms of the antenna separation and azimuth. Let the antenna separation be D, the azimuth angle be θ, and the difference in antenna heights above the reference ellipse be ΔH.

The in-plane distance between the antenna locations (D_{plane}) is:

$$D_{plane} = (D^2 - \Delta H^2)^{\frac{1}{2}}\ .$$

From Figs. 9 and 10 and the preceding definitions, it can be shown that:

$$\Delta X = -D_{plane}\,(\sin\varphi_n \cos\lambda_n \cos\theta_n + \sin\lambda_n \sin\theta_n) + \Delta H \cos\varphi_n \cos\lambda_n \qquad (13)$$

$$\Delta Y = -D_{plane}\,(\sin\varphi_n \sin\lambda_n \cos\theta_n - \cos\lambda_n \sin\theta_n) + \Delta H \cos\varphi_n \sin\lambda_n \qquad (14)$$

$$\Delta Z = D_{plane} \cos\varphi_n \cos\theta_n + \Delta H \sin\varphi_n\ . \qquad (15)$$

These expressions for ΔX, ΔY, and ΔZ can be substituded back into Eq. (12), giving :

$$\begin{aligned}\Delta SR = \frac{D_{plane}}{SR}\{[(X_{net}\cos\lambda + Y_{net}\sin\lambda)\sin\varphi - Z_{net}\cos\varphi]\cos\theta \\ + (X_{net}\sin\lambda - Y_{net}\cos\lambda)\sin\theta\} \\ - \frac{\Delta H}{SR}[(X_{net}\cos\lambda + Y_{net}\sin\lambda)\cos\varphi + Z_{net}\sin\varphi]\ . \qquad (16)\end{aligned}$$

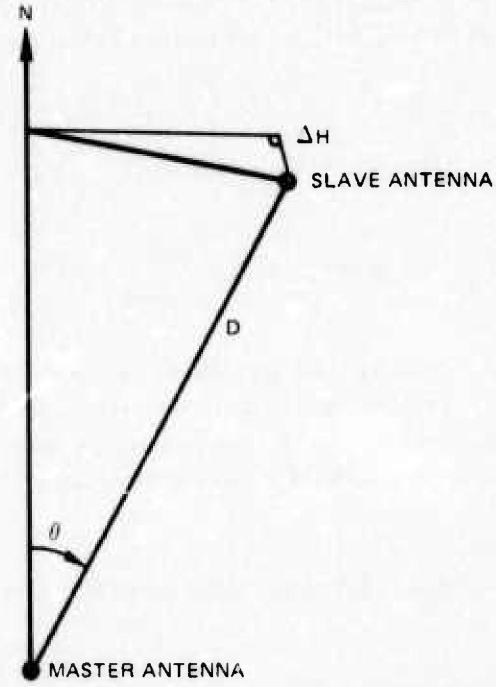

Fig. 9 MASTER-SLAVE ANTENNA IN-PLANE GEOMETRY

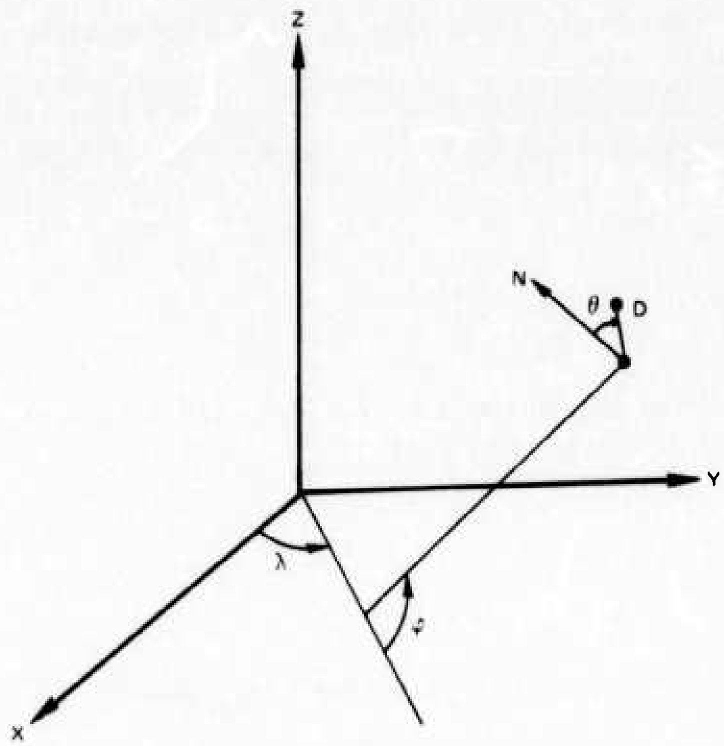

Fig. 10 EARTH-REFERENCED MASTER-SLAVE ANTENNA GEOMETRY

The path length difference between the two antennas (ΔSR) can be expressed in wavelengths as $N\lambda$, where λ is the wavelength of the frequency being received and N is the number of cycles. If another measurement were taken at a later time, $\Delta SR_2 = N_2\lambda$.

It is not convenient to determine the actual value of N. Therefore, the change in phase with time is determined by measuring the phase change between times t_1 and t_2. This gives:

$$\Delta SR_2 - \Delta SR_1 = (N_2 - N_1)\lambda \quad .$$

The process is repeated for as many time intervals as possible during a satellite pass in a manner similar to the satellite location technique. Using this method, it never becomes necessary to know the original N_1; knowledge is required of only the phase differences.

APPLICATIONS

There are a variety of solution forms to the problem, depending on what information is known and what is needed, as listed below:

Possible Application	Known Variables	Solution Variables	Minimum Required Number of Intervals
Directions in earth-fixed coordinates	None	ΔX, ΔY, ΔZ	3
for aligning navigation platforms	D	ΔX, ΔZ or ΔX, ΔY or ΔY, ΔZ	2
Azimuth determination	ΔH	Azimuth (θ), D	2
	ΔH, D	Azimuth (θ)	1

Because the quantities being measured in each interval are not exact, but are corrupted by noise, it is desirable to obtain more data intervals than required. It will be shown in a later section how dramatically the results improve with increasing number of data points.

Assume that one wishes to solve for ΔX, ΔY, ΔZ in earth-fixed coordinates. Also assume that there were a total of m intervals during which phase changes were measured and the positions of the satellite at the fiducial marks were known.

The solution of the problem involves an iterative least squares fit to the data; therefore, initial estimates of ΔX, ΔY, and ΔZ are necessary. From these, estimates of ΔSR can also be obtained.

To recall Eq. (12):

$$\Delta SR = - \frac{1}{SR} (X_{net} \Delta X + Y_{net} \Delta Y + Z_{net} \Delta Z) \ .$$

Therefore:

$$\frac{\partial (\Delta SR)}{\partial (\Delta X)} = - \frac{X_{net}}{SR}$$

$$\frac{\partial (\Delta SR)}{\partial (\Delta Y)} = - \frac{Y_{net}}{SR}$$

$$\frac{\partial (\Delta SR)}{\partial (\Delta Z)} = - \frac{Z_{net}}{SR}$$

For the m intervals, a mean square error may be defined by:

$$F(\Delta X, \Delta Y, \Delta Z) = \sum_{k=1}^{m} [N_k \lambda - (\Delta SR_k - \Delta SR_{k-1})]^2 \ .$$

In order to obtain a least squares fit, it is required that:

$$\frac{\partial F}{\partial (\Delta X)} = \frac{\partial F}{\partial (\Delta Y)} = \frac{\partial F}{\partial (\Delta Z)} = 0 \ .$$

To solve these equations, a C matrix may be constructed from the initial estimates of ΔX, ΔY, and ΔZ; the measured values of N; and known positions of the navigator and satellite:

$$C(k, 1) = N_k \lambda - (\Delta SR_k - \Delta SR_{k-1})$$

$$C(k, 2) = \frac{\partial (\Delta SR)_k}{\partial (\Delta X)} - \frac{\partial (\Delta SR)_{k-1}}{\partial (\Delta X)}$$

$$C(k, 3) = \frac{\partial (\Delta SR)_k}{\partial (\Delta Y)} - \frac{\partial (\Delta SR)_{k-1}}{\partial (\Delta Y)}$$

$$C(k, 4) = \frac{\partial (\Delta SR)_k}{\partial (\Delta Z)} - \frac{\partial (\Delta SR)_{k-1}}{\partial (\Delta Z)} \ ,$$

giving a C matrix of the form:

$$\{C\} = \begin{bmatrix} C(1,1) & C(1,2) & C(1,3) & C(1,4) \\ C(2,1) & . & . & . \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ C(M,1) & C(M,2) & C(M,3) & C(M,4) \end{bmatrix}$$

From the C matrix a new matrix is defined whose terms are:

$$A_{ij} = \sum_{k=1}^{m} C_{ki} \cdot C_{kj} \ ,$$

where

$i = 2, 3, 4$, and

$j = 1, 2, 3, 4$.

This gives three simultaneous equations:

$$A_{21} + A_{22}\, d(\Delta X) + A_{23}\, d(\Delta Y) + A_{24}\, d(\Delta Z) = 0$$

$$A_{31} + A_{32}\, d(\Delta X) + A_{33}\, d(\Delta Y) + A_{34}\, d(\Delta Z) = 0$$

$$A_{41} + A_{42}\, d(\Delta X) + A_{43}\, d(\Delta Y) + A_{44}\, d(\Delta Z) = 0 \ .$$

From these three equations, $d(\Delta X)$, $d(\Delta Y)$, and $d(\Delta Z)$ can be found.

If a new matrix is defined:

$$B_{11} = A_{33} A_{44} - A_{43} A_{34}$$

$$B_{21} = A_{23} A_{44} - A_{43} A_{24}$$

$$B_{31} = A_{23} A_{34} - A_{33} A_{24}$$

$$B_{12} = A_{32} A_{44} - A_{42} A_{34}$$

$$B_{22} = A_{22} A_{44} - A_{42} A_{24}$$

$$B_{32} = A_{22} A_{34} - A_{32} A_{24}$$

$$B_{13} = A_{32} A_{43} - A_{42} A_{33}$$

$$B_{23} = A_{22} A_{43} - A_{42} A_{23}$$

$$B_{33} = A_{22} A_{33} - A_{32} A_{23} \ ,$$

and:

$$E = A_{22} B_{11} - A_{32} B_{21} + A_{42} B_{31} \ ,$$

then:

$$d(\Delta X) = \frac{1}{E} (-A_{21} B_{11} + A_{31} B_{21} - A_{41} B_{31})$$

$$d(\Delta Y) = \frac{1}{E} (A_{21} B_{12} - A_{31} B_{22} + A_{41} B_{32})$$

$$d(\Delta Z) = \frac{1}{E} (-A_{21} B_{13} + A_{31} B_{23} - A_{41} B_{33}) \ .$$

The estimates of ΔX, ΔY, and ΔZ are then updated:

$$\Delta X_{new} = \Delta X_{old} + d(\Delta X)$$

$$\Delta Y_{new} = \Delta Y_{old} + d(\Delta Y)$$

$$\Delta Z_{new} = \Delta Z_{old} + d(\Delta Z).$$

If $|d(\Delta X)|$, $|d(\Delta Y)|$, and $|d(\Delta Z)|$ are all less than a predetermined breakout criterion, then:

$$\Delta X_{fix} = \Delta X_{new}$$

$$\Delta Y_{fix} = \Delta Y_{new}$$

$$\Delta Z_{fix} = \Delta Z_{new} \quad .$$

Otherwise, the new values of ΔX, ΔY, and ΔZ are used as improved estimates and the iteration process is repeated until the breakout criterion is satisfied.

Now assume that the antenna separation D is known by some other method (such as surveying) to a high degree of accuracy. From the relationship:

$$D = [(\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2]^{\frac{1}{2}} \quad ,$$

it can be seen that only two independent variables exist in this solution. As an example, assume that ΔY and ΔZ will be found. Then:

$$\Delta X = [D^2 - (\Delta Y)^2 - (\Delta Z)^2]^{\frac{1}{2}}$$

$$\Delta SR = -\frac{1}{SR}\left\{X_{net}[D^2 - (\Delta Y)^2 - (\Delta Z)^2]^{\frac{1}{2}} + Y_{net}\,\Delta Y + Z_{net}\,\Delta Z\right\}$$

$$\frac{\partial(\Delta SR)}{\partial(\Delta Y)} = -\frac{1}{SR}\left\{Y_{net} - \frac{X_{net}(\Delta Y)}{[D^2 - (\Delta Y)^2 - (\Delta Z)^2]^{1/2}}\right\}$$

$$\frac{\partial(\Delta SR)}{\partial(\Delta Z)} = -\frac{1}{SR}\left\{Z_{net} - \frac{X_{net}(\Delta Z)}{[D^2 - (\Delta Y)^2 - (\Delta Z)^2]^{1/2}}\right\} .$$

The Taylor series expansion of $F(\Delta Y, \Delta Z)$ then becomes:

$$F(\Delta Y, \Delta Z) = \sum_{k=1}^{m}\left[N_k \lambda - (\Delta SR_k - \Delta SR_{k-1}) - \frac{\partial(\Delta SR_k - \Delta SR_{k-1})}{\partial(\Delta Y)} d(\Delta Y) - \frac{\partial(\Delta SR_k - \Delta SR_{k-1})}{\partial(\Delta Z)} d(\Delta Z)\right]^2 ,$$

and the solution proceeds in a similar manner:

$$C(k, 1) = N_k \lambda - (\Delta SR - \Delta SR_{k-1})$$

$$C(k, 2) = \frac{\partial(\Delta SR)_k}{\partial(\Delta Y)} - \frac{\partial(\Delta SR)_{k-1}}{\partial(\Delta Y)}$$

$$C(k, 3) = \frac{\partial(\Delta SR)_k}{\partial(\Delta Z)} - \frac{\partial(\Delta SR)_{k-1}}{\partial(\Delta Z)}$$

$$A_{ij} = \sum_{k=1}^{m} C_{ki} C_{kj} ,$$

where

$i = 2, 3,$ and

$j = 1, 2, 3.$

$$A_{21} + A_{22}\, d(\Delta Y) + A_{23}\, d(\Delta Z) = 0$$

$$A_{31} + A_{32}\, d(\Delta Y) + A_{33}\, d(\Delta Z) = 0$$

$$d(\Delta Y) = \frac{(A_{31} A_{23} - A_{21} A_{33})}{(A_{22} A_{33} - A_{32} A_{23})}$$

$$d(\Delta Z) = \frac{(A_{21} A_{32} - A_{31} A_{22})}{(A_{22} A_{33} - A_{32} A_{23})}$$

$$\Delta Y_{new} = \Delta Y_{old} + d(\Delta Y)$$

$$\Delta Z_{new} = \Delta Z_{old} + d(\Delta Z) \ .$$

If $|d(\Delta Y)|$ or $|d(\Delta Z)|$ are greater than the predetermined breakout criterion, then the iteration is repeated. Otherwise,

$$\Delta Z_{fix} = \Delta Z_{new}$$

$$\Delta Y_{fix} = \Delta Y_{new}$$

$$\Delta X_{fix} = [D^2 - (\Delta Y)^2_{fix} - (\Delta Z)^2_{fix}]^{\frac{1}{2}} \ .$$

AZIMUTH DETERMINATION

Recalling Eq. (16):

$$\Delta SR = - [(X_{net} \cos \lambda + Y_{net} \sin \lambda) \cos \varphi + Z_{net} \sin \varphi] \frac{\Delta H}{SR}$$
$$+ \frac{D_{plane}}{SR} \{[(X_{net} \cos \lambda + Y_{net} \sin \lambda) \sin \varphi - Z_{net} \cos \varphi] \cos \theta$$
$$+ [X_{net} \sin \lambda - Y_{net} \cos \lambda] \sin \theta\}$$

From the expression for ΔSR, the partials with respect to D_{plane} and θ can be computed:

$$\frac{\partial(\Delta SR)}{\partial(D_{plane})} = \frac{1}{SR}\{[(X_{net}\cos\lambda + Y_{net}\sin\lambda)\sin\varphi - Z_{net}\cos\varphi]\cos\theta$$

$$+ [X_{net}\sin\lambda - Y_{net}\cos\lambda]\sin\theta\} \ , \tag{17}$$

$$\frac{\partial(\Delta SR)}{\partial(\theta)} = \frac{D_{plane}}{SR}\{[X_{net}\sin\lambda - Y_{net}\cos\lambda]\cos\theta$$

$$- [(X_{net}\cos\lambda + Y_{net}\sin\lambda)\sin\varphi - Z_{net}\cos\varphi]\sin\theta\} \ . \tag{18}$$

If we make the following definition:

$$C(k, 1) = N_k\lambda - (\Delta SR_k - \Delta SR_{k-1})$$

$$C(k, 2) = \frac{\partial(\Delta SR)_k}{\partial(D_{plane})} - \frac{\partial(\Delta SR)_{k-1}}{\partial(D_{plane})}$$

$$C(k, 3) = \frac{\partial(\Delta SR)_k}{\partial(\theta)} - \frac{\partial(\Delta SR)_{k-1}}{\partial(\theta)} \ ,$$

it follows that:

$$A_{ij} = \sum_{k=1}^{m} C_{ki} \cdot C_{kj} \ ,$$

where

$$i = 2, \ 3$$

$$j = 1, \ 2, \ 3$$

$$A_{21} + A_{22}\, d(D_{plane}) + A_{23}\, d(\theta) = 0$$

$$A_{31} + A_{32}\, d(D_{plane}) + A_{33}\, d(\theta) = 0 \ .$$

Solving for $d(D_{plane})$ and $d(\theta)$:

$$d(D_{plane}) = \frac{[A_{31} A_{23} - A_{21} A_{33}]}{[A_{22} A_{33} - A_{32} A_{23}]}$$

$$d(\theta) = \frac{[A_{21} A_{32} - A_{31} A_{22}]}{[A_{22} A_{33} - A_{32} A_{23}]}$$

$$D_{plane\ new} = D_{plane\ old} + d(D_{plane})$$

$$\theta_{new} = \theta_{old} + d(\theta) \quad .$$

If $|d(D_{plane})|$ or $|d(\theta)|$ are greater than the predetermined breakout criterion, then the iteration is repeated.

Otherwise:

$$D_{plane\ fix} = D_{plane\ new}$$

$$\theta_{fix} = \theta_{new} \quad .$$

If the antenna separation (D) is known, then the calculation is somewhat simplified and only the azimuth angle is a variable of solution.

Again from Eq. (18):

$$\frac{\partial(\Delta SR)}{\partial(\theta)} = \frac{D_{plane}}{SR} \{[X_{net} \sin\lambda - Y_{net} \cos\lambda] \cos\theta$$
$$-[(X_{net} \cos\lambda + Y_{net} \sin\lambda) \sin\varphi - Z_{net} \cos\varphi] \sin\theta\} \quad .$$

The function F is now only dependent on θ:

$$F(\theta) = \sum_{k=1}^{m} [N_k \lambda - (\Delta SR_k - \Delta SR_{k-1}) - \frac{\partial(\Delta SR_k - \Delta SR_{k-1})}{\partial(\theta)} d(\theta)]^2 \quad .$$

The C matrix then becomes:

$$C(k, 1) = N_k\lambda - (\Delta SR_k - \Delta SR_{k-1})$$

$$C(k, 2) = \frac{\partial (\Delta SR)_k}{\partial (\theta)} - \frac{\partial (\Delta SR)_{k-1}}{\partial (\theta)} \; .$$

From which:

$$A_1 = \sum_{k=1}^{m} C_{k2} \cdot C_{k1}$$

$$A_2 = \sum_{k=1}^{m} C_{k2}{}^2$$

$$A_1 + A_2 \, d(\theta) = 0 \; .$$

Therefore:

$$d(\theta) = -A_1/A_2 \; .$$

The estimate of θ is then improved:

$$\theta_{new} = \theta_{old} + d(\theta) \; .$$

If $|d(\theta)|$ is less than the breakout criterion, then $\theta_{fix} = \theta_{new}$; otherwise the iteration repeated.

3. HARDWARE DEVELOPMENT AND INITIAL TESTS

INITIAL TEST CONFIGURATION

The initial Aztran tests employed two APL translocation backpacks as satellite receivers. These units (Fig. 11) received the 400 MHz satellite signals on antennas separated by about 50 meters. Both receivers were driven by the same 5 MHz local oscillator to provide phase coherence between the two 32 kHz intermediate frequency outputs.

The phase difference between the two 32 kHz outputs was measured by the phase comparator. A flip-flop was turned on by the negative-going zero crossing of one 32 kHz IF and turned off by the negative-going zero crossing of the other. The flip-flop's output was then integrated over many cycles producing an analog voltage proportional to the phase shift between the IF signals and fed to a strip chart recorder.

Results

A number of satellite passes were received using this configuration. The phase was recorded on a strip chart and the 2 minute time marks were superimposed over the phase. To determine the number of phase counts during a 2 minute interval, the charted sawtooth wave forms were counted between the time markers and recorded.

From a survey, the azimuth of the antenna base line was determined and compared to the computed results. The errors in various passes are plotted in Fig. 12 against the number of usable 2 minute data intervals received. As can be seen, the errors were strongly dependent on the number of intervals, indicating a great deal of noise in the data.

Many intervals had to be discarded because one or the other of the receivers would occasionally lose the satellite signal. If this happened for even 1 second then the data from the entire 2 minute interval had to be discarded. This initial test configuration demonstrated the system concept:

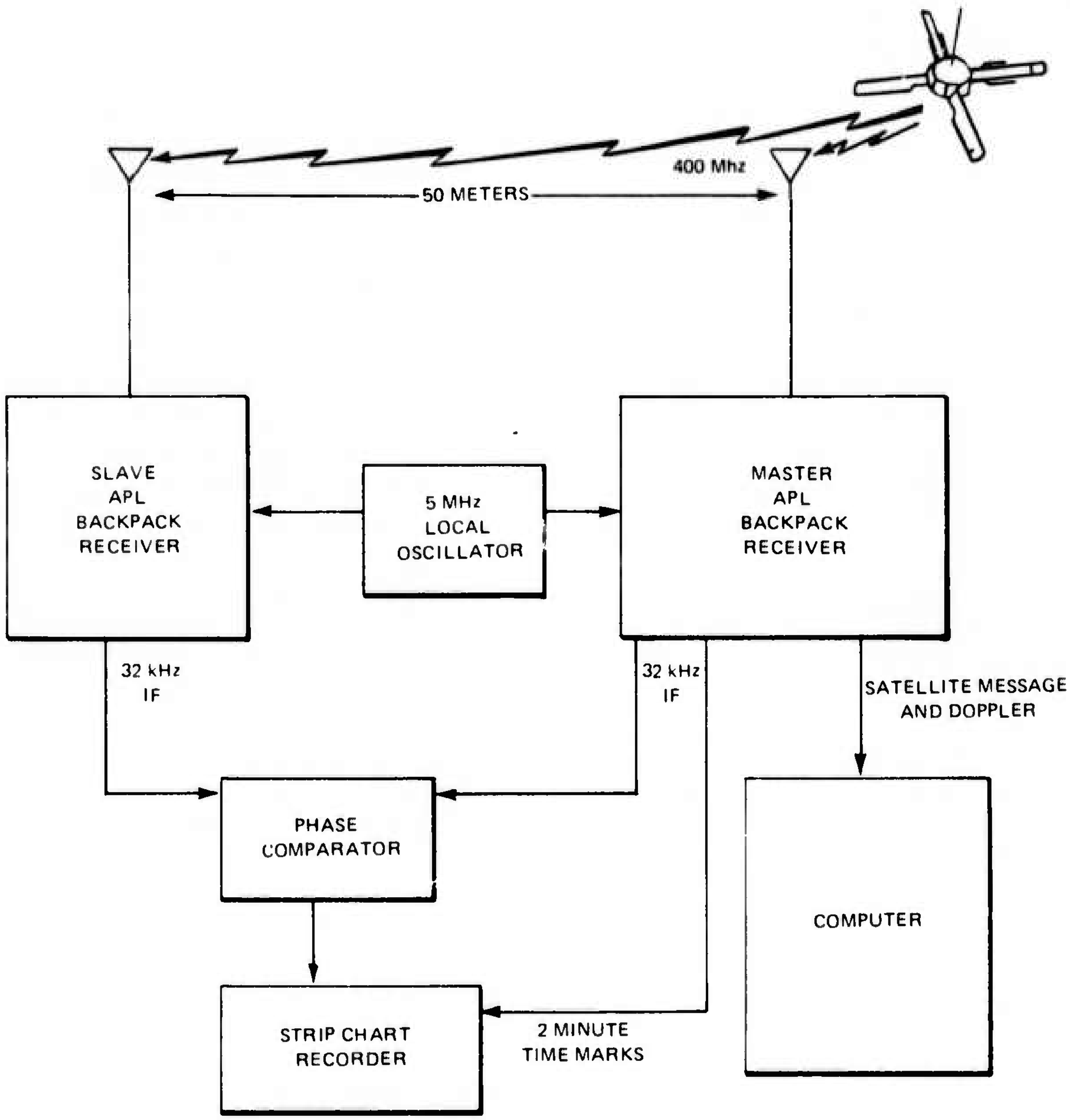

Fig. 11 SETUP FOR INITIAL AZTRAN TESTS

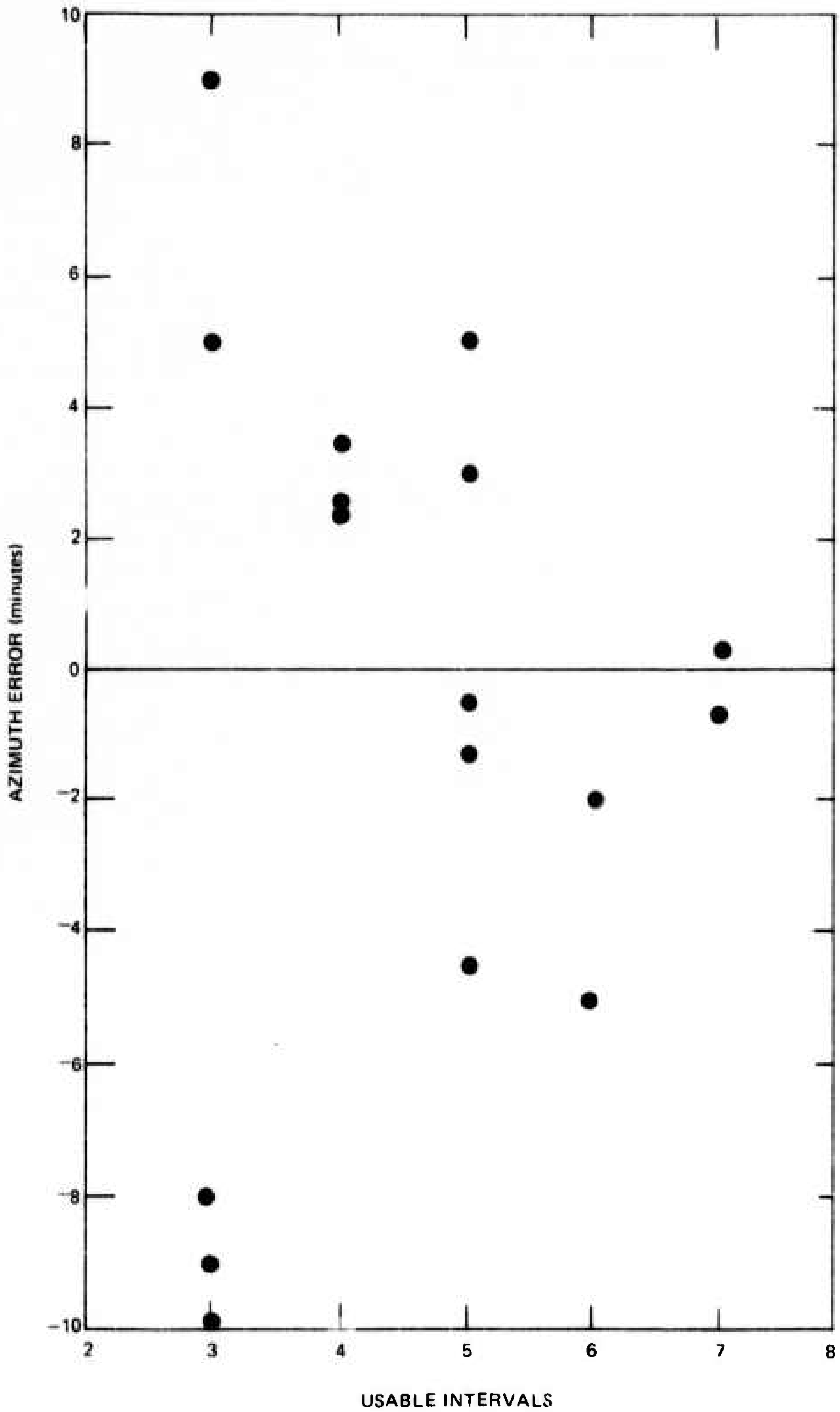

Fig. 12 PASS AZIMUTH ERRORS VERSUS NUMBER OF USABLE 2 MINUTE INTERVALS

however, it became apparent that if more accuracy were desired, better use would have to be made of the available data.

IMPROVED TEST CONFIGURATION

There were two basic problem areas in the initial test configuration. The first problem was the large amount of usable data which was being discarded when the receivers would lose the signal for a short interval. This problem was minimized by changing the measurement intervals from 2 minutes to 15 seconds. Thus only 15 seconds of data would be lost if a receiver were to momentarily lose the signal. Shortening the measurement interval required a knowledge of the satellite position every 15 seconds instead of every 2 minutes. These coordinates were obtained by a fourth order Lagrangian interpolation of the 2 minute interval X-Y-Z coordinates.

The second problem involved data reduction. In the original system, it was necessary to manually read the strip chart recording and estimate the true phase from the noisy phase recording. It became desirable to automate the entire process, which would allow data collection on a 24 hour basis. To solve this problem, use was made of an available Honeywell H-21 computer. This is an 8k computer with 18-bit words which was already interfaced to the APL backpack receiver to perform the navigation calculations.

A different phase comparator was built that gave a digital measure of the phase difference in the signal. On command from the computer, one counter would count the number of 10 MHz clock pulses occurring in one complete cycle of one of the 32 kHz IF outputs. Another counter would simultaneously count the number of 10 MHz clock pulses occurring between negative-going zero crossings of the two IF outputs. These two counts were transferred to the computer and a ratio of them taken giving the percentage phase delay to an accuracy of about 2°.

The computer sampled the phase up to 100 times per second and then fed these samples through a second order

digital filter. The filter predicted the next phase measurement and the error in the prediction was used to generate a correlation value. When a significant difference between the measured and predicted phase appeared, it was assumed that one or both of the receivers had lost the signal and the particular data interval was disregarded.

The value of the filter's output was stored every 15 seconds and used in the Aztran calculations. Figure 13 is a flow chart of the steps involved in generating the filtered phase output.

During a satellite pass the computer stores the satellite message and the doppler counts used for navigation. Simultaneously the computer digitally filters the measured phase and stores the output of the filter every 15 seconds. At the conclusion of the satellite pass, the computer executes the navigation calculations and prints the results. The stored values from the digital filter for every 15 seconds during the pass and the satellite's X-Y-Z coordinates are then punched on paper tape for later processing. When a roll of paper tape is accumulated (usually in about 3 days), it is transferred to magnetic tape and used as an input to the Aztran calculations program. This PL/I program (listed in Appendix A) computes the azimuth and separation, or just the azimuth if the antenna separation is known.

Figure 14 shows Aztran equipment installed in a van. On the left is the teletype that prints the results from the computer, which is to it right. In the equipment rack (top to bottom) are the phase measuring interface, paper tape reader and punch, and one of the two satellite receivers. On the table to the right is a strip chart recorder and the second receiver.

Figure 15 is a photo of one of the two Aztran antennas. It is a 10 foot tall volute wound for 400 MHz. This antenna was designed to have uniform coverage in the upper hemisphere and attenuate sharply any reflections from below the horizon in order to reduce phase errors.

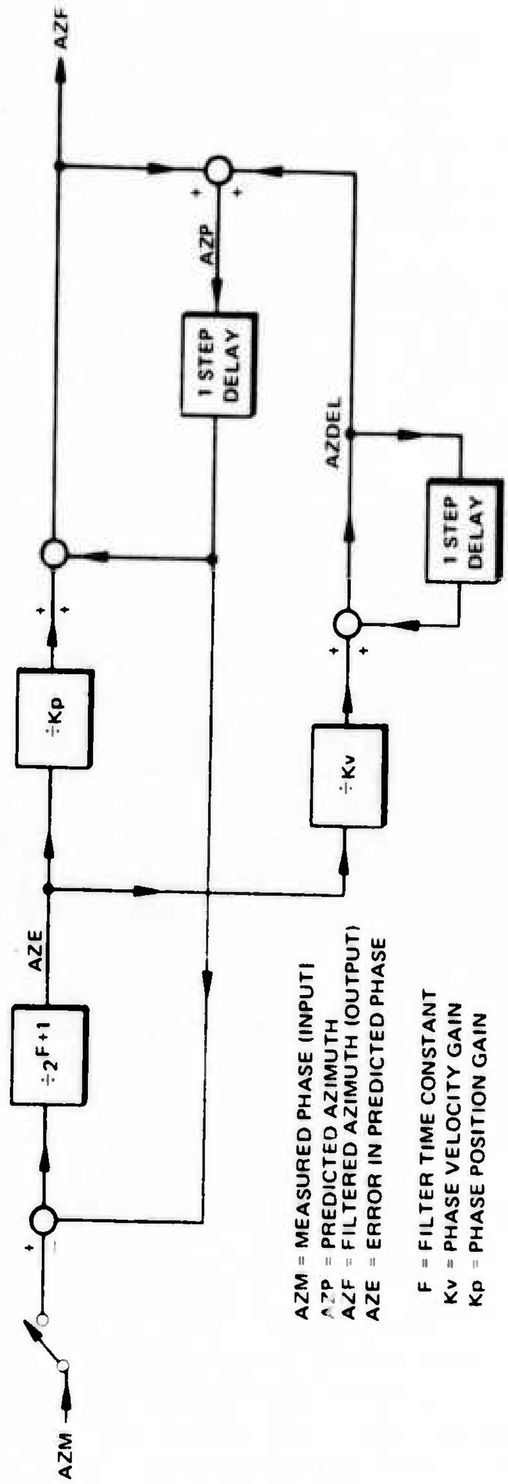

Fig. 13 AZTRAN SECOND ORDER DIGITAL FILTER

Fig. 14 AZTRAN EQUIPMENT

Fig. 15 AZTRAN-VOLUTE ANTENNA

Results

Tests were run using the improved test configuration for various filter parameters listed in Fig. 13. From the tests, it was determined that the following appeared to be optimum values:

$$K_v = 0.8$$

$$K_p = 0.03 \quad ,$$

with a sample rate of 12 samples per second.

With these values fixed, groups of passes were received and analyzed for different values of the constant FSHIFT. Table 1 lists the results. In part A, it can be seen that the rms error in azimuth and separation is fairly constant for values of FSHIFT less than 9. In this calculation, azimuth and separation were both variables of the solution. In the azimuth solution, one of the groups of passes was analyzed solving only for azimuth. The separation was determined by a survey. The rms error in this instance was about half that obtained in the azimuth and separation solution. This again indicates that there is a great deal of noise in the data since most of these passes had six or seven good 2 minute intervals.

Table 1

Results of Computer Filtered
Aztran Passes

AZIMUTH AND SEPARATION SOLUTION

FSHIFT	Antenna Azimuth		Antenna Separation (meters)		Number of Passes
	Mean	rms	Mean	rms	
6	73°, 0.316'	1.63'	51.899	0.072	6
7	73°, 0.094'	1.64'	51.846	0.115	9
8	73°, 0.463'	1.69'	51.868	0.099	16
9	73°, 1.907'	6.834'	51.829	0.214	13

AZIMUTH SOLUTION

FSHIFT	Antenna Azimuth		Number of Passes
	Mean	rms	
8	73°, 0.705'	0.88'	17

4. CONCLUSIONS

The tests run to data have given very encouraging initial results. The theory and system concept have been proven to be practical. If the system errors can be further reduced to about 20" of arc (a factor of three improvement), then the system can compete with north-seeking gyro compasses in the equatorial and mid-latitude regions.

Multipath errors from ground reflections and tracking filter errors appears to be the dominant problems. An error analysis of the Aztran system (Appendix B) indicates that improved accuracies should be achievable if the phase measurement errors can be brought under control and the antenna separation accurately measured. As the system exists, without improvements, it is still an unique tool for polar navigation offering the only true all weather azimuth measuring system which exhibits no singularities at or near the poles.

ACKNOWLEDGMENT

The author would like to express his appreciation to J. A. Ford for his design of the phase measuring interface, which is the core of the Aztran system, and to E. E. Westerfield for his advice and help.

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Appendix A

AZTRAN COMPUTER PROGRAM

The following is a listing of the PL/I program used to solve for antenna azimuth and separation. The inputs to the program include navigator's position, satellite position as a function of time, estimates of azimuth and antenna separation, and the measurements of phase change during the satellite pass.

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THE JOHNS HOPKINS UNIVERSITY
APPLIED PHYSICS LABORATORY
SILVER SPRING MARYLAND

PL/I OPTIMIZING COMPILER   AZTRAN: PROCEDURE OPTIONS (MAIN) REORDER;

SOURCE LISTING

```
STMT LEV NT

   1      0  |AZTRAN: PROCEDURE CPTIONS (MAIN( RECRDER:
             |/*       AZTRAN NAVIGATION PROGRAM*/
   2   1  0  | DCL (ONSOURCE,ONCHAR| BUILTIN;
   3   1  0  |DCL(XNET(0:64(,YNET(0:64|,ZNET(0:64)| FLCAT(15);
   4   1  0  |DCL(XS(C:64|,YS(D:64),ZS(0:64|| FLCAT(15(;
   5   1  0  (DCL(SR(0:64|,DSR(0:64|,DSRD(0:64|,DSRAZ(0:64|) FLOAT(15);
   6   1  0  |DCL(C(0:63,3),A(2:3,3(| FLOAT(15);
   7   1  0  |DCL(XN,YN,ZN,BLATN,BLONG,E,H,G) FLCAT(15|;
   8   1  0  |DCL PHASE(0:63|,TEST(0:63),D(0:63),DEL(0:63|;
   9   1  0  |DCL (DA(75(,TI(75|,AZID(75|,AZIM(75)) INIT(0);
  10   1  0  (DCL (SEF(75|,INT(75|,AZIMD(75),AZIMM(75|) INIT(0);
  11   1  0  |DCL NOGCOD CHAR(80|,BAD CHAR(3(;
  12   1  0  |DCL (AZ,AZACT,AZD,AZER,AZEST,CACT,DAZ) FLOAT(15);
  13   1  0  |DCL (DCPLAN,DELTH,CEST,DPLAN,F,HT,R,RE,WAVE) FLOAT(15).
             |         /*          */
  14   1  0  |         RE=6378140;              /*EARTH'S RADIUS IN METERS*/
  15   1  0  |         WAVE=.74948234;          /*WAVELENGTH AT 400MHZ. (METERS) */
  16   1  0  |         F=0.003353229;           /*EARTH'S COEFFICIENT OF FLATTENING*/
  17   1  0  (         BREAK=.0001;             /*BREAKCUT LIMIT*/
  18   1  0  |         P=1;                     /* PASS NUMBER*/
             |         /*          */
  19   1  0  |ON ENDFILE(TAPE1| BEGIN;
  20   2  0  |         PUT SKIP LIST('END CF FILE');
  21   2  0  |         GO TO ND;
  22   2  0  |    END;
  23   1  0  |ON CCNVERSICN BEGIN;
  24   2  0  (         PUT SKIP LIST(ONCHAR,CNSOURCE);
  25   2  0  (         DC I=1 TO 10;
  26   2  1  (         GET FILE(TAPE1| SKIP EDIT(NOGOOD)(A(80));
  27   2  1  |         PUT SKIP LIST(NOGCOO|;
  28   2  1  |         END;
  29   2  0  |         BLANK=0;
  30   2  0  (         GO TO LCAD;
  31   2  0  |    END;
             |         /*          */
             |         /*          */
  32   1  C  ;GET LIST(AZACT,AZEST,DACT,DEST,DELTH|;
             |    /*AZACT=SURVEYED AZIMUTH,USE 0 IF UNKNCWN*/
             |    /*AZEST=ESTIMATED AZIMUTH*/
             |    /*AZACT & AZEST IN DEGREES AND FRACTICNS*/
             |    /*DACT=SURVEYED SEPARATION, USE 0 IF UNKNOWN*/
             |    /*DEST=ESTIMATEC SEPARATICN*/
             (    /*DELTH=HT OF REMOTE ANTENNA ABOVE MASTER*/
             (    /*ALL CISTANCES IN METERS*/
             (         /*          */
  33   1  0  (GET LIST(BLATN,BLONG,HT);
```


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```
PL/I OPTIMIZING COMPILER         AZTRAN: PROCEDURE OPTIONS (MAIN) REORDER;

    STMT LEV NT

                    |    /*BLATH=SURVEYED LATITUDE IN DEGREES OF MASTER ANT*/
                    |    /*BLONG=SURVEYED LONGITUDE IN DEGREES OF MASTER ANT*/
                    |    /*HT=HT IN METERS OF MASTER ANT ABOVE GEOID*/
     34    1   0    |    GET LIST(THRESH);
                    |          /*         */
     35    1   0    |RESTART: XS=0;
     36    1   0    |          YS=0;
     37    1   0    |          ZS=0;
     38    1   0    |          ZAF=0;
     39    1   0    |          BLANK=0;
                    |          /*         */
                    |    /*ROUTINE TO SEARCH FOR START OF PASS ON MAG TAPE*/
                    |          /*         */
     40    1   0    |LOAD: GET FILE(TAPE1) EDIT(BAD) (A(3));
     41    1   0    |      IF SUBSTR(BAD,1,1)=' ' THEN DO;
     42    1   1    |           BLANK=BLANK+1;
     43    1   1    |           GET FILE(TAPE1) SKIP(1);
     44    1   1    |           GO TO LOAD;
     45    1   1    |           END;
     46    1   0    |      ELSE DO;
     47    1   1    |           IF BLANK>7  THEN DO;
     48    1   2    |                DDAY=BAD;
     49    1   2    |                GET FILE(TAPE1) EDIT(TTIME) (X(3),F(4));
     50    1   2    |                BLANK=0;
     51    1   2    |                GO TO OK;
     52    1   2    |                END;
     53    1   1    |           ELSE DO;
     54    1   2    |                BLANK=0;
     55    1   2    |                GET FILE(TAPE1) SKIP(1);
     56    1   2    |                GO TO LOAD;
     57    1   2    |                END;
     58    1   1    |           END;
     59    1   0    |OK:  DA(P)=DDAY;
     60    1   0    |      TI(P)=TTIME;
                    |          /*         */
                    |     /* READ IN THE XYZS*/
                    |          /*         */
     61    1   0    |DO I=0 TO 56 BY 8;
     62    1   1    |   GET FILE(TAPE1) SKIP EDIT(XS(I),YS(I),ZS(I))
                    |      (E(13,8),X(3),E(13,8),X(3),E(13,8));
     63    1   1    |END;
                    |          /*         */
                    |    /*READ PHASE AND CORRELATION*/
                    |          /*         */
     64    1   0    |DO I=0 TO 56;
     65    1   1    |GET FILE(TAPE1) SKIP EDIT(PHASE(I),D(I)) (F(7),X(3),F(6));
     66    1   1    |END;
                    |    /*SCALE PHASE*/
     67    1   0    |PHASE=PHASE/1000;
```

```
PL/I OPTIMIZING COMPILER        AZTRAN: PROCEDURE OPTIONS (MAIN) REORDER;

  STMT LEV NT

                  |    /*SEARCH FOR LAST STORED PHASE INTERVAL (=DD)*/                   |
    68   1   0    |DO I=63 TO 0 BY -1;                                                   |
    69   1   1    |    IF D(I)>D THEN GO TO D1;                                          |
    70   1   1    |END;                                                                  |
    71   1   0    |D1: DD=8*TRUNC(I/8);                                                  |
                  |         /*          */                                               |
                  |    /*FIND GOOD PHASE INTERVALS AND COMPUTE DIFFERENCES*/             |
                  |         /*          */                                               |
    72   1   0    |DO I=DD TO 2 BY -1;                                                   |
    73   1   1    |    IF (D(I)>THRESH|D(I)=0) THEN DO;                                  |
    74   1   2    |         PHASE(I)=999;                                                |
    75   1   2    |         ZAP=ZAP+1;                                                   |
    76   1   2    |         END;                                                         |
    77   1   1    |    ELSE PHASE(I)=PHASE(I-1)-PHASE(I);                                |
    78   1   1    |    IF ABS(PHASE(I))>15 THEN PHASE(I)=999;                            |
    79   1   1    |END;                                                                  |
    80   1   0    |PHASE(0)=999;                                                         |
    81   1   0    |PHASE(1)=999;                                                         |
    82   1   0    |IF DD-ZAP<20 THEN GO TO ENDD;                                         |
    83   1   0    |INT(P)=DD-ZAP;                                                        |
                  |         /*          */                                               |
                  |    /*INTERPOLATE FROM 2 MIN INTERVALS TO 15 SEC INTERVALS*/          |
                  |         /*          */                                               |
    84   1   0    |CALL INTERP(XS, DD);                                                  |
    85   1   0    |CALL INTERP(YS, DD);                                                  |
    86   1   0    |CALL INTERP(ZS, DD);                                                  |
                  |         /*          */                                               |
                  |    /*COMPUTE NAVIGATOR'S POSITION*/                                  |
                  |         /*          */                                               |
    87   1   0    | R=RE/SQRT(COSD(BLATN)**2+(1-F)**2*SIND(BLATN)**2);                   |
    88   1   0    |XN=(R+HT)*COSD(BLATN)*COSD(BLONG);                                    |
    89   1   0    |YN=(R+HT)*COSD(BLATN)*SIND(BLONG);                                    |
    90   1   0    |ZN=(R*(1-F)**2+HT)*SIND(BLATN);                                       |
                  |         /*          */                                               |
                  |    /*COMPUTE SLANT RANGES*/                                          |
                  |         /*          */                                               |
    91   1   0    |DO K=0 TO DD;                                                         |
    92   1   1    |    XNET(K)=XS(K)-XN;                                                 |
    93   1   1    |    YNET(K)=YS(K)-YN;                                                 |
    94   1   1    |    ZNET(K)=ZS(K)-ZN;                                                 |
    95   1   1    |    SR(K)=SQRT(XNET(K)**2+YNET(K)**2+ZNET(K)**2);                     |
    96   1   1    |END;                                                                  |
                  |         /*          */                                               |
                  |    /*COMPUTE THEORITICAL PHASE*/                                     |
                  |         /*          */                                               |
    97   1   0    |DEL=D;                                                                |
    98   1   0    |TEST=0;                                                               |
    99   1   0    |IF AZACT=0 THEN GO TO ZZ1;                                            |
   100   1   0    |IF DACT=0 THEN GO TO ZZ1;                                             |
```

```
PL/I OPTIMIZING COMPILER        AZTRAN: PROCEDURE OPTIONS (MAIN) REORDER;

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   101   1  0  |DPLAN=SQRT(DACT**2-DELTH**2);
   102   1  0  |AZ=AZACT;
               |        /*         */
               |   /*COMPUTE PARTIALS*/
               |        /*         */
   103   1  0  |DO K=0 TO DD;
   104   1  1  |   E=-(XNET(K)*COSD(BLONG)+YNET(K)*SIND(BLONG))*SIND(BLATN)
               |         +ZNET(K)*COSD(BLATN);
   105   1  1  |   H=((XNET(K)*COSD(BLONG)+YNET(K)*SIND(BLONG))*COSD(BLATN)
               |         +ZNET(K)*SIND(BLATN));
   106   1  1  |   G=-XNET(K)*SIND(BLONG)+YNET(K)*COSD(BLONG);
   107   1  1  |   DSR(K)=DPLAN*(E*COSD(AZ)+G*SIND(AZ))/SR(K)+H*DELTH/SR(K);
   108   1  1  |END;
   109   1  0  |TEST(0)=999;
   110   1  0  |DO K=1 TO DD;
   111   1  1  |   TEST(K)=(DSR(K-1)-DSR(K))/WAVE;
   112   1  1  |   DEL(K)=PHASE(K)-TEST(K);
   113   1  1  |   IF PHASE(K)=999 THEN DEL(K)=0;
   114   1  1  |END;
               |        /*         */
   115   1  0  |ZZ1: PUT SKIP EDIT('DAY ',DDAY,'    TIME ',ZTIME, 'GMT')
               |(A,F(3),A,F(4),A);
   116   1  0  |PUT SKIP(2);
   117   1  0  |PUT SKIP LIST(' K          XS              YS            ZS      M.PHASE
               |   T.PHASE     DELTA PHASE     CORRELATION     ELEV');
   118   1  0  |DO K=0 TO DD;
   119   1  1  | Z1= SQRT(XN**2+YN**2+ZN**2);
   120   1  1  | Z2=SQRT(XS(K)**2+YS(K)**2+ZS(K)**2);
   121   1  1  | Z3=(Z2**2-Z1**2-SR(K)**2)/(2*Z1*SR(K));
   122   1  1  | EL=ATAND(Z3/SQRT(1-Z3**2));
   123   1  1  |PUT SKIP EDIT(K,XS(K),YS(K),ZS(K),PHASE(K),TEST(K),DEL(K),D(K),EL)
               |  (F(2),F(14,2),F(14,2),F(14,2),F(11,4),X(3),F(8,4),X(5),F(8,4),
               |        X(8),F(8),X(7),F(5,1));
   124   1  1  |END;
   125   1  0  |PUT PAGE;
               |        /*         */
               |        /*         */
               |   /*SOLVE FOR AZIMUTH AND SEPARATION*/
               |        /*         */
   126   1  0  |PUT LIST('AZIMUTH AND ANTENNA SEPARATION ITERATION');
   127   1  0  |PUT SKIP(3);
   128   1  0  |AZ=AZEST;
   129   1  0  |DPLAN=SQRT(DEST**2-DELTH**2);
   130   1  0  |I=0;
               |        /*         */
               |   /*COMPUTE PARTIALS*/
               |        /*         */
   131   1  0  | START:  DO K=0 TO DD;
   132   1  1  |   E=-(XNET(K)*COSD(BLONG)+YNET(K)*SIND(BLONG))*SIND(BLATN)
```

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PL/I OPTIMIZING COMPILER        AZTRAN: PROCEDURE OPTIONS (MAIN) REORDER;

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                     |          +ZNET(K)*CCSD(BLATN);
     133   1   1     |    H=((XNET(K)*COSD(BLONG)+YNET(K)*SIND(BLONG))*COSD(B'ATN)
                     |          +ZNET(K)*SIND(BLATN));
     134   1   1     |    G=-XNET(K)*SIND(BLONG)+YNET(K)*COSD(BLONG);
     135   1   1     |    DSRD(K)=(E*COSD(AZ)+G*SIND(AZ))/SR(K);
     136   1   1     |    DSRAZ(K)=DFLAN*(-E*SIND(AZ)+G*COSD(AZ))/SR(K);
     137   1   1     |    DSR(K)=DFLAN*(E*COSD(AZ)+G*SIND(AZ))/SR(K)+H*DELTH/SR(K);
     138   1   1     |END;
                     |          /*          */
                     |    /*COMPUTE C MATRIX*/
                     |          /*          */
     139   1   0     |A=0;
     140   1   0     |C=C;
     141   1   0     |DO K=0 TO DD-1;
     142   1   1     |IF PHASE(K+1)=999 THEN GO TO OUT;
     143   1   1     |C(K,1)=PHASE(K+1)*WAVE+DSR(K+1)-DSR(K);
     144   1   1     |C(K,2)=DSRAZ(K+1)-DSRAZ(K);
     145   1   1     |C(K,3)=DSRD(K+1)-DSRD(K);
     146   1   1     |    DO M=2 TO 3;
     147   1   2     |          DC J=1 TO 3;
     148   1   3     |                A(M,J)=A(M,J)+C(K,M)*C(K,J);
     149   1   3     |          END;
     150   1   2     |    END;
     151   1   1     |OUT: END;
                     |          /*          */
                     |    /*COMPUTE AND TEST RESIDUALS*/
                     |          /*          */
     152   1   0     |DAZ=(A(2,3)*A(3,1)-A(2,1)*A(3,3))/(A(2,2)*A(3,3)-A(2,3)*A(3,2));
     153   1   0     |DAZ=5[illegible]8*DAZ;
     154   1   0     |DDP[illegible],1)*A(2,2)-A(2,1)*A(3,2))/(A(2,3)*A(3,2)-A(2,2)*A(3,3));
     155   1   0     |DP[illegible]+DDPLAN;
     156   1   0     |AZ=A[illegible]
     157   1   0     |I=I+1;
     158   1   0     |IF I>20 THEN GO TO ENDD;     /*TOO MANY ITERATIONS*/
     159   1   0     |IF ABS(DAZ)>BREAK THEN GO TO START;
     160   1   0     |IF ABS(DDPLAN)>0.1 THEN GC TC START;
                     |          /*          */
     161   1   0     |SIGMA=0;
     162   1   0     |DDD=DD;
     163   1   0     |PUT EDIT('K          RESIDUAL            ITERATION NUMB.R',I)(A,F(3));
     164   1   0     |DO K=0 TO DD-1;
     165   1   1     |PUT SKIP EDIT(K,C(K,1))(F(2),X(5),F(14,4));
     166   1   1     |      SIGMA=SIGMA+(C(K,1))**2;
     167   1   1     |      IF C(K,1)=0 THEN DDD=DDD-1;
     168   1   1     |END;
     169   1   0     |SIGMA=SQRT(SIGMA/DDD);
     170   1   0     |PUT SKIP(2);
     171   1   0     |AZD=TRUNC(AZ);
     172   1   0     |AZID(P)=AZD;
```

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PL/I OPTIMIZING COMPILER        AZTRAN: PROCEDURE OPTIONS (MAIN) REORDER;

    STMT LEV NT

     173   1  0  |AZM=60*ABS(AZ-AZD);
     174   1  0  |AZIM(P)=AZM;
     175   1  0  |PUT SKIP EDIT('AZIMUTH      ',AZD,' DEG    ',AZM,' MIN',
                 |     (A,F(3),A,F(7,4),A);
     176   1  0  |PUT SKIP EDIT('SEPARATION      ',SQRT(DPLAN**2+DELTH**2),' METERS')
                 |     (A,F(7,3),A);
     177   1  0  |SEP(I)=SQRT(DPLAN**2+DELTH**2);
     178   1  0  |PUT SKIP;
     179   1  0  |IF AZACT=0 THEN GO TO ZZ2;
     180   1  0  |AZER=60*(AZACT-AZ);
     181   1  0  |PUT SKIP EDIT('AZIMUTH ERROR      ',AZER,' MIN') (A,F(9,7),A);
     182   1  0  |ZZ2:IF DACT=0 THEN GO TO ZZ7;
     183   1  0  |PUT SKIP EDIT('SEP. ERROR    ',DACT-SQRT(DPLAN**2+DELTH**2),' METERS')
                 |     (A,F(7,4),A);
                 |        /*          */
                 |    /*TEST FOR 3 SIGMA RESIDUALS*/
                 |        /*          */
     184   1  0  |ZZ7:  JJ=0;
     185   1  0  |DO K=0 TO DD-1;
     186   1  1  |IF ABS(C(K,1))>3*SIGMA THEN DO;
     187   1  2  |    PHASE(K+1)=999;
     188   1  2  |    JJ=JJ+1;
     189   1  2  |END;
     190   1  1  |END;
     191   1  0  |PUT SKIP EDIT('3 SIGMA STRIP=',3*SIGMA) (A,F(6,4));
     192   1  0  |PUT SKIP(2);
     193   1  0  |IF JJ>0 THEN GO TO START;
     194   1  0  |IF DACT=0 THEN GO TO TALLY;
     195   1  0  |PUT PAGE;
                 |        /*          */
                 |        /*          */
                 |    /*AZIMUTH ITERATION,MUST KNOW DACT*/
                 |        /*          */
     196   1  0  |AZIT: PUT SKIP LIST('AZIMUTH ITERATION');
     197   1  0  |I=0;
     198   1  0  |PUT SKIP(4);
     199   1  0  |DPLAN=SQRT(DACT**2-DELTH**2);
     200   1  0  |AZ=AZEST;
                 |        /*          */
                 |    /*COMPUTE PARTIALS*/
                 |        /*          */
     201   1  0  |BEGIN: DO K=0 TO DD;
     202   1  1  |    E=-(XNET(K)*COSD(BLONG)+YNET(K)*SIND(BLONG))*SIND(BLATN)
                 |          +ZNET(K)*COSD(BLATN);
     203   1  1  |    H=((XNET(K)*COSD(BLONG)+YNET(K)*SIND(BLONG))*COSD(BLATN)
                 |          +ZNET(K)*SIND(BLATN))*DELTH/DPLAN;
     204   1  1  |    G=-XNET(K)*SIND(BLONG)+YNET(K)*COSD(BLONG);
     205   1  1  |    DSRAZ(K)=DPLAN*(-E*SIND(AZ)+G*COSD(AZ))/SR(K);
     206   1  1  |    DSR(K)=DPLAN*(H+E*COSD(AZ)+G*SIND(AZ))/SR(K);
```

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PL/I OPTIMIZING COMPILER        AZTRAN: PROCEDURE OPTIONS (MAIN) REORDER;

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     207   1  1  |DSR(K)=DPLAN*(H+E*COSD(AZ)+G*SIND(AZ))/SR(K);                                  |
     208   1  1  |END;                                                                           |
                 |       /*          */                                                          |
                 |   /*COMPUTE C MATRIX*/                                                        |
                 |       /*          */                                                          |
     209   1  0  |A=0;                                                                           |
     210   1  0  |C=0;                                                                           |
     211   1  0  |DO K=0 TO DD-1;                                                                |
     212   1  1  |IF PHASE(K+1)=999 THEN GO TO OUTT;                                             |
     213   1  1  |C(K,1)=PHASE(K+1)*WAVE+DSR(K+1)-DSR(K);                                        |
     214   1  1  |C(K,2)=DSRAZ(K+1)-DSRAZ(K);                                                    |
     215   1  1  |A(2,1)=A(2,1)+C(K,1)*C(K,2);                                                   |
     216   1  1  |A(2,2)=A(2,2)+C(K,2)**2;                                                       |
     217   1  1  |OUTT: END;                                                                     |
                 |       /*          */                                                          |
                 |   /*COMPUTE AND TEST RESIDUALS*/                                              |
                 |       /*          */                                                          |
     218   1  0  |DAZ=-57.29578*A(2,1)/A(2,2);                                                   |
     219   1  0  |AZ=AZ+DAZ;                                                                     |
     220   1  0  |I=I+1;                                                                         |
     221   1  0  |IF I>20 THEN GO TO TALLY;                                                      |
     222   1  0  |IF ABS(DAZ)>BREAK THEN GO TO BEGIN;                                            |
                 |       /*          */                                                          |
     223   1  0  |SIGMA=0;                                                                       |
     224   1  0  |DDD=DD;                                                                        |
     225   1  0  |PUT SKIP(1);                                                                   |
     226   1  0  | PUT EDIT('K           RESIDUAL            ITERATION NUMBER',I) (A,F(3));        |
     227   1  0  |DO K=0 TO DD-1;                                                                |
     228   1  1  |PUT SKIP EDIT(K,C(K,1)) (F(2),X(5),F(14,4));                                   |
     229   1  1  |     SIGMA=SIGMA+(C(K,1))**2;                                                   |
     230   1  1  |     IF C(K,1)=0 THEN DDD=DDD-1;                                                |
     231   1  1  |END;                                                                           |
     232   1  0  |SIGMA=SQRT(SIGMA/DDD);                                                         |
     233   1  0  |PUT SKIP(2);                                                                   |
     234   1  0  |AZD=TRUNC(AZ);                                                                 |
     235   1  0  |AZIMD(P)=AZD;                                                                  |
     236   1  0  |AZM=60*ABS(AZ-AZD);                                                            |
     237   1  0  |AZIMM(P)=AZM;                                                                  |
     238   1  0  |AZER=60*(AZACT-AZ);                                                            |
     239   1  0  |PUT SKIP EDIT('AZIMUTH     ',AZD,' DEG    ',AZM,' MIN')                          |
                 |    (A,F(3),A,F(7,4),A);                                                       |
     240   1  0  |IF AZACT=0 THEN GO TO ZZ8;                                                     |
     241   1  0  |PUT SKIP EDIT('AZIMUTH ERROR     ',AZER,' MIN') (A,F(9,4),A);                  |
                 |       /*          */                                                          |
                 |   /*TEST FOR 3 SIGMA RESIDUALS*/                                              |
                 |       /*          */                                                          |
     242   1  0  |ZZ8: JJ=0;                                                                     |
     243   1  0  |DO K=0 TO DD-1;                                                                |
     244   1  1  |IF ABS(C(K,1))>3*SIGMA THEN DO;                                                |
```

```
PL/I CPTIMIZING COMPILER        AZTRAN: PROCEDURE OPTIONS (MAIN) REORDER;

    STMT LEV NT

     245   1  2  |    PHASE(K+1)=999;
     246   1  2  |    JJ=JJ+1;
     247   1  2  |END;
     248   1  1  |END;
     249   1  0  |PUT SKIP EDIT('3 SIGMA STRIF=',3*SIGMA) (A,F(6,4));
     250   1  0  |PUT SKIF(2);
     251   1  0  |IF JJ>0 THEN GC TC BEGIN;
     252   1  0  |TALLY: P=P+1;
     253   1  0  |IF P>75 THEN P=75;
     254   1  0  |ENDC: PUT PAGE;
     255   1  0  |GO TO RESTART;
                 |        /*          */
                 |        /*          */
                 |    /*LAGRANGIAN INTERPOLATION ROUTINE*/
                 |    /*THE CRDER OF INTERPOLATICN VARIES WITH THE NUMBER OF INTERVALS*/
                 |        /*          */
     256   1  0  |INTERP: PROCEDURE(ARRAY, CC) REORDER;
     257   2  0  |DCL ARRAY(0:64) FLCAT(15) CONNECTED;
     258   2  0  |    DO M=0 TO  CC-8 BY 8;
     259   2  1  |        DO K=M+1 TO M+7;
     260   2  2  |            IF M>8 THEN L=M-8;
     261   2  2  |            IF M>CC-16 THEN L=CC-24;
     262   2  2  |            IF M<16 THEN L=0;
     263   2  2  |            DO I=L TO L+24 BY 8;
     264   2  3  |                TERM=1;
     265   2  3  |                IF I=L THEN DC;
     266   2  4  |                    DO J=L+8 TO L+24 BY 8;
     267   2  5  |                    TERM=TERM*(K-J)/(I-J);
     268   2  5  |                    END;
     269   2  4  |                    END;
     270   2  3  |                ELSE DO;
     271   2  4  |                    DO J=L TO I-8 BY 8,I+8 TO L+24 BY 8;
     272   2  5  |                    TERM=TERM*(K-J)/(I-J);
     273   2  5  |                    END;
     274   2  4  |                    END;
     275   2  3  |                ARRAY(K)=ARRAY(K)+TERM*ARRAY(I);
     276   2  3  |            END;
     277   2  2  |        END;
     278   2  1  |    END;
     279   2  0  |END INTERP;
     280   1  0  |ND: PUT PAGE;
     281   1  0  |ON ERROR SYSTEM;
     282   1  0  |PUT EDIT('LAT ',BLATN,'LON ',BLONG,'HT ',HT)
                 |        (A,F(9,4),X(5),A,F(9,4),X(5),A,F(7,2));
     283   1  0  |PUT SKIF EDIT('AZACT ',AZACT,'DACT ',DACT,'DH ',DELTH)
                 |        (A,F(9,4),X(5),A,F(7,2),X(5),A,F(7,3));
     284   1  0  |    PUT SKIP EDIT('THRESH ',THRESH)(A,F(5));
     285   1  0  |PUT SKIF(2);
     286   1  0  |PUT SKIF EDIT('DAY    TIME    INT    AZIMUTH         SEPARATION    AZIMUTH
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PL/I OPTIMIZING COMPILER        AZTRAN: PROCEDURE OPTIONS (MAIN) REORDER;

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                 |ONLY'( (A);                                                       (
     287   1  0  (PUT SKIP EDIT('DEG MIN','METERS','DEG MIN')(X(19),A,X(10),A,X(7),A);  (
     288   1  0  |PUT SKIP(2);                                                       |
     289   1  0  (DO I=1 TO P-1;                                                     |
     290   1  1  |PUT SKIP EDIT(DA(I),TI(I),INT(I),AZID(I),AZIM(I),SEP(I),AZIMD(I),   (
                 |AZIMM(I))(F(3),F(7),F(6),F(7),F(8,4),F(11,3),F(9),F(8,4));          (
     291   1  1  (END;                                                               (
     292   1  0  (FIN01:  END AZTRAN;                                                |
```

Appendix B

AZTRAN ERROR MODEL

SUMMARY

A mathematical error model of the Aztran system has been developed to study the limiting accuracies of the system. Reasonable measurement uncertainties were selected, and used as inputs to the model. Based on these uncertainties, a 50 meter baseline system should be able to achieve an accuracy of about 50" to 60" of arc rms and a 100 meter baseline system should approach 25" of arc rms in the mid latitudes. The values vary somewhat with the azimuth angle of the antenna and the pass geometry.

Uncertainties of 20 meters rms in latitude and longitude and 4 meters in height were used for the navigator's position. This degree of accuracy is obtainable by averaging a small group of refraction corrected passes. The error in the navigator's position contributes almost nothing to the total system error and could be 10 to 100 times greater in magnitude before it approached the value of some of the other errors.

An rms value of 8 meters in positional uncertainty was assumed for the satellite in each of the three Cartesian coordinates at any one fiducial mark. This value of 8 meters is combined satellite orbit prediction error and H-21 computer conversion error from Keplerian to Cartesian elements. No error correlation was assumed from measurement to measurement. The position errors contribute little to the system error and could be as much as a factor of 10 larger before they would begin to become significant.

Two different pairs of uncertainties were used for estimates of the antenna separation and height difference. For the first pair 3 cm and 1 cm were used, respectively. At this measurement accuracy, these uncertainties tended

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to be the dominant error source for a 100 meter baseline system.

The second set of uncertainties were 1 cm in separation and 0.5 cm in height difference. These values require considerably more precise measurement techniques but are still feasible. In this case the antenna measurement errors were significant for a 50 meter baseline system, however in a 100 meter baseline system the dominant error source was the phase measurement error.

In the current Aztran system the phase is sampled 100 times per second by the computer in a second order digital filter. The maximum phase frequency being followed is about 1 Hz. Sources of phase error are jitter in the receiver phase-locked loops, multipath reflections, and sampling error. A value of 0.01 cycle of phase error was selected as the rms measurement error. This is not to imply an error of 0.01 cycle per phase sample, but that the smoothed output from the digital filter (which is stored every 15 seconds) will have this rms error. Due to the high sampling rate (1500 samples between stored values), the error from stored value to stored value was assumed to be uncorrelated.

The second section is a development of the equations used in the error model and the final section discusses the results of the model in more detail and their implications.

ERROR ANALYSIS EQUATIONS

In the Aztran system, a signal path length difference exists between the two antennas. This difference is determined by measuring the phase difference in the two received signals:

$$\Delta SR = N\lambda, \qquad \text{(B-1)}$$

where:

ΔSR is the slant range difference,

N is the number of wavelengths, and

λ is the wavelength of the received signal.

Recalling Eq. (12) of this report:

$$\Delta SR = (X \cdot \Delta X + Y \cdot \Delta Y + Z \cdot \Delta Z), \tag{B-2}$$

where, in Cartesian coordinates:

$$X = X_{satellite} - X_{navigator} = X_s - X_n$$

$$Y = Y_{satellite} - Y_{navigator} = Y_s - Y_n$$

$$Z = Z_{satellite} - Z_{navigator} = Z_s - Z_n \ .$$

From Eq. (10) of this report:

$$D = (\Delta X^2 + \Delta Y^2 + \Delta Z^2)^{\frac{1}{2}} \ , \tag{B-3}$$

where:

D = the antenna separation and

$$D_{plane} = \sqrt{D^2 - (\Delta H)^2} \ .$$

From Eqs. (13) through (15) of this report:

$$\Delta X = - D_{plane} [\sin \varphi \ \cos \lambda \ \cos \theta + \sin \lambda \ \sin \theta] + \Delta H \cos \varphi \ \cos \lambda \tag{B-4}$$

$$\Delta Y = - D_{plane} [\sin \varphi \ \sin \lambda \ \cos \theta - \cos \lambda \ \sin \theta] + \Delta H \ \cos \varphi \ \sin \lambda \tag{B-5}$$

$$\Delta Z = D_{plane} \ \cos \varphi \ \cos \theta + \Delta H \ \sin \varphi \ , \tag{B-6}$$

where:

φ = navigator's latitude,

λ = navigator's longitude,

θ = baseline azimuth angle, and

ΔH = antenna height difference.

From the preceding equations, ΔSR can be defined as:

$$\Delta SR = f(\varphi, \lambda, HT, X_s, Y_s, Z_s, D_{plane}, \Delta H, \theta) \quad (B-7)$$

The true ΔSR can be expressed in terms of a Taylor expansion about the nominal value:

$$\Delta SR_t = \Delta SR_n + \frac{\partial f}{\partial \varphi}(\varphi_t - \varphi_n) + \frac{\partial f}{\partial \lambda}(\lambda_t - \lambda_n) + \cdots\cdots$$

$$= \Delta SR_n + \frac{\partial f}{\partial \varphi}\delta\varphi + \frac{\partial f}{\partial \lambda}\delta\lambda + \frac{\partial f}{\partial HT}\delta HT + \cdots\cdots$$

The measured ΔSR can be expressed in terms of the true value and the measured phase error:

$$\Delta SR_m = \Delta SR_n + \frac{\lambda}{360}\,\delta\,(ph_m)\,.$$

By subtracting the nominal value from the measured value, the true value drops out:

$$\Delta SR_m - \Delta SR_n = \frac{\lambda}{360}\,\delta\,(ph_m) - \frac{\partial f}{\partial \theta}\delta\theta - \left|\frac{\partial f}{\partial \varphi}\;\frac{\partial f}{\partial \lambda}\;\cdots\;\frac{\partial f}{\partial \Delta H}\right| \begin{bmatrix} \delta\varphi \\ \delta\lambda \\ \cdot \\ \cdot \\ \cdot \\ \delta\Delta H \end{bmatrix} . \quad (B-8)$$

During a satellite pass, many measurements are made. An estimated azimuth angle is then improved in an iterative least squares sense to drive the nominal ΔSR to the same value as the measured ΔSR. Equation (B-8) now becomes:

$$\frac{\partial f}{\partial \theta} \delta\theta = \frac{\lambda}{360} \delta(ph_m) - \left[\frac{\partial f}{\partial \varphi} \cdot \cdot \cdot \cdot \frac{\partial f}{\partial \Delta H}\right] \begin{bmatrix} \delta\varphi \\ \delta\lambda \\ \cdot \\ \cdot \\ \cdot \\ \delta\Delta H \end{bmatrix} \quad . \quad \text{(B-9)}$$

In a similar manner, the antenna separation error sensitivity can be determined:

$$\frac{\partial f}{\partial D} \delta D = \frac{\lambda}{360} \delta(ph_m) - \left[\frac{\partial f}{\partial \varphi} \frac{\partial f}{\partial \lambda} \cdot \cdot \cdot \frac{\partial f}{\partial \Delta H} \frac{\partial f}{\partial \theta}\right] \begin{bmatrix} \delta\theta \\ \delta\lambda \\ \cdot \\ \cdot \\ \cdot \\ \delta\Delta H \\ \delta\theta \end{bmatrix} \quad . \quad \text{(B-10)}$$

Assume that during the satellite pass, n separate measurements are made, each representing a data interval. Eq. (B-9) now takes the form

$$\begin{bmatrix} \frac{\partial f_1}{\partial \theta} \\ \frac{\partial f_2}{\partial \theta} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial f_n}{\partial \theta} \end{bmatrix} \delta\theta = \frac{\lambda}{360} \begin{bmatrix} \delta(ph_m)_1 \\ \delta(ph_m)_2 \\ \cdot \\ \cdot \\ \cdot \\ \delta(ph_m)_n \end{bmatrix} - \begin{bmatrix} \frac{\partial f_1}{\partial \varphi} & \frac{\partial f_1}{\partial \lambda} & \cdots & \frac{\partial f_1}{\partial \Delta H} \\ \frac{\partial f_2}{\partial \varphi} & \frac{\partial f_2}{\partial \lambda} & \cdots & \frac{\partial f_2}{\partial \Delta H} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \frac{\partial f_n}{\partial \varphi} & \frac{\partial f_n}{\partial \lambda} & \cdots & \frac{\partial f_n}{\partial \Delta H} \end{bmatrix} \begin{bmatrix} \delta\varphi \\ \delta\lambda \\ \cdot \\ \cdot \\ \delta\Delta H \end{bmatrix} \cdot \quad \text{(B-11)}$$

Let Eq. (B-11) be represented in matrix notation as:

$$\underline{R} \cdot \delta\theta = \frac{\lambda}{360} \cdot \underline{M} - \underline{K} \cdot \underline{ER} \ . \qquad \text{(B-12)}$$

Using Eq. (B-12) and estimates of the various errors (φ, λ, ..., ΔH, δph_m) an error in $\delta\theta$ can be determined to be

$$\delta\theta = \underline{R}^t \cdot \left(\frac{\lambda}{360} \cdot \underline{M} - \underline{K} \cdot \underline{ER}\right) \cdot \left(\frac{1}{R^t\ R}\right) \ . \qquad \text{(B-13)}$$

However, the desired result is not the error from a single pass for a given set of biases but the statistical variance for a pass given an estimated variance in the error sources.

This becomes:

$$[\delta\theta]^2 = \underline{R}^t \cdot \left\{\left(\frac{\lambda}{360}\right)^2 [\underline{M} \cdot \underline{M}^t] + \underline{K} \cdot [\underline{ER} \cdot \underline{ER}^t] \cdot \underline{K}^t\right\} \cdot \underline{R} \cdot \left(\frac{1}{R^t\ R}\right)^2 \qquad \text{(B-14)}$$

In Eq. (B-14) $[\underline{M} \cdot \underline{M}^t]$ is the measurement error variance matrix which is diagonalized, implying no correlation from measurement to measurement. Similarly $[\underline{ER} \cdot \underline{ER}^t]$ is the diagonalized error variance measurement, again assuming no correlation from error source to error source.

From Eq. (B-14) the system sensitivities to each variable can be derived and a total error budget can be determined. The only remaining problem is to calculate the partial derivatives of f. Recalling Eq. (B-2):

$$\Delta SR = -\frac{1}{SR} \cdot (X \cdot \Delta X + Y \cdot \Delta Y + Z \cdot \Delta Z) \ .$$

Therefore, the partial derivative of ΔSR can be written as:

$$\frac{\partial \Delta SR}{\partial i} = -\frac{\partial(1/SR)}{\partial i} (X \cdot \Delta X + Y \cdot \Delta Y + Z \cdot \Delta Z)$$

$$-\frac{1}{SR} \cdot \left(\frac{\partial X}{\partial i} \Delta X + X \frac{\partial \Delta X}{\partial i} + \frac{\partial Y}{\partial i} \Delta Y + Y \frac{\partial \Delta Y}{\partial i} + \frac{\partial Z}{\partial i} \Delta Z + Z \frac{\partial \Delta Z}{\partial i}\right) \ . \qquad \text{(B-15)}$$

where i is one of the variables $\varphi, \lambda, X_s, Y_s, Z_s, D, \Delta H, \theta$. Recalling Eq. (9) from this report:

$$1/SR = (X^2 + Y^2 + Z^2)^{-\frac{1}{2}} \; ,$$

so that:

$$\frac{\partial (1/SR)}{\partial i} = -\frac{1}{SR^3} \left(X \frac{\partial X}{\partial i} + Y \frac{\partial Y}{\partial i} + Z \frac{\partial Z}{\partial i} \right) \; , \qquad \text{(B-16)}$$

$$X = X_s - X_n \; ,$$

$$X = X_s - (R + H_T) \cos\varphi \; \cos\lambda \; , \qquad \text{(B-17)}$$

where:

$$R = Re \cdot [\cos^2 \varphi + (1-f)^2 \; \sin^2 \varphi]^{-\frac{1}{2}} \; ,$$

Re = equatorial earth radius,

φ = navigator's latitude,

λ = navigator's longitude, and

H_T = navigator's height above the reference ellipsoid.

From Eq. (B-17) it can be seen that:

$$\frac{\partial X}{\partial X_s} = 1$$

$$\frac{\partial X}{\partial H_T} = -\cos\varphi \; \cos\lambda$$

$$\frac{\partial X}{\partial \varphi} = (R + H_T) \; \sin\varphi \; \cos\lambda$$

$$+ \frac{R^3}{Re^2} \cdot f \cdot (f-2) \cdot \cos^2\varphi \; \sin\varphi \; \cos\lambda$$

$$\frac{\partial X}{\partial \lambda} = (R + H_T) \cos \phi \sin \lambda \quad .$$

$$Y = Y_s - Y_n$$

$$Y = Y_s - (R + H_T) \cos \phi \sin \lambda \quad . \tag{B-18}$$

Therefore:

$$\frac{\partial Y}{\partial Y_s} = 1$$

$$\frac{\partial Y}{\partial H_T} = - \cos \phi \sin \lambda$$

$$\frac{\partial Y}{\partial \phi} = (R + H_T) \sin \phi \sin \lambda + \frac{R^3}{Re^2} \cdot f \cdot (f-2) \cdot \cos^2 \phi \sin \phi \sin \lambda$$

$$\frac{\partial Y}{\partial \lambda} = - (R + H_T) \cos \phi \cos \lambda \quad .$$

$$Z = Z_s - Z_n$$

$$Z = Z_s - \left[R(1-f)^2 + H_T \right] \sin \phi \tag{B-19}$$

$$\frac{\partial Z}{\partial Z_s} = 1$$

$$\frac{\partial Z}{\partial H_T} = - \sin \phi$$

$$\frac{\partial Z}{\partial \varphi} = -\left[R(1-f)^2 + H_T\right] \cos\varphi$$

$$+ \frac{R^3}{Re^2} \cdot (1-f)^2 \cdot f \cdot (f-2) \sin^2\varphi \cos\varphi \ .$$

$$\Delta X = -D_{plane} (\sin\varphi \cos\lambda \cos\theta + \sin\lambda \sin\theta)$$

$$+ \Delta H \cos\varphi \cos\lambda \qquad \text{(B-20)}$$

$$\frac{\partial \Delta X}{\partial D_{plane}} = -\sin\varphi \cos\lambda \cos\theta - \sin\lambda \sin\theta$$

$$\frac{\partial \Delta X}{\partial \theta} = D_{plane} (\sin\varphi \cos\lambda \sin\theta - \sin\lambda \cos\theta)$$

$$\frac{\partial \Delta X}{\partial \Delta H} = \cos\varphi \cos\lambda$$

$$\frac{\partial \Delta X}{\partial \varphi} = -D_{plane} \cos\varphi \cos\lambda \cos\theta - \Delta H \sin\varphi \cos\lambda$$

$$\frac{\partial \Delta X}{\partial \lambda} = D_{plane} (\sin\varphi \sin\lambda \cos\theta - \cos\lambda \sin\theta)$$

$$- \Delta H \cos\varphi \sin\lambda \ .$$

$$\Delta Y = -D_{plane} (\sin\varphi \sin\lambda \cos\theta - \cos\lambda \sin\theta)$$

$$+ \Delta H \cos\varphi \sin\lambda \ . \qquad \text{(B-21)}$$

$$\frac{\partial \Delta Y}{\partial D_{plane}} = -\sin\varphi \sin\lambda \cos\theta + \cos\lambda \sin\theta$$

$$\frac{\partial \Delta Y}{\partial \theta} = D_{plane} (\sin\varphi \sin\lambda \sin\theta + \cos\lambda \cos\theta)$$

$$\frac{\partial \Delta Y}{\partial \Delta H} = \cos\varphi \sin\lambda$$

$$\frac{\partial \Delta Y}{\partial \varphi} = - D_{plane} \cos\varphi \ \sin\lambda \ \cos\theta$$

$$- \Delta H \sin\varphi \ \sin\lambda$$

$$\frac{\partial \Delta Y}{\partial \lambda} = - D_{plane} (\sin\varphi \ \cos\lambda \ \cos\theta + \sin\lambda \ \sin\theta)$$

$$+ \Delta H \cos\varphi \ \cos\lambda \ .$$

$$\Delta Z = D_{plane} \ \cos\varphi \ \cos\theta + \Delta H \sin\varphi \qquad \text{(B-22)}$$

$$\frac{\partial \Delta Z}{\partial D_{plane}} = \cos\varphi \ \cos\theta$$

$$\frac{\partial \Delta Z}{\partial \theta} = - D \ \cos\varphi \ \sin\theta$$

$$\frac{\partial \Delta Z}{\partial \Delta H} = \sin\varphi$$

$$\frac{\partial \Delta Z}{\partial \varphi} = - D_{plane} \ \sin\varphi \ \cos\theta + \Delta H \ \cos\varphi \ .$$

RESULTS

The results of the error analysis for various passes has been tabulated to show the total rms error and the contribution from each error source (in seconds of arc) for antenna azimuths from 0° to 90°. The pass geometry is described for each pass. The error values chosen were 20 meters in latitude and longitude, 4 meters in height, 8 meters in the satellite's X, Y, and Z coordinates and 0.01 λ in measurement noise.

Table B-1 shows the results of a few passes over APL's Howard County facility with a 100 meter baseline for antenna measurement errors of 1 cm in separation and 0.5 cm in height difference. Table B-2 shows the same passes

over APL with a 50 meter baseline and antenna measurement errors of 1 cm in separation and 0.5 cm in height difference. As can be seen by comparing Tables B-1 and B-2, azimuth error due to errors in antenna separation, height difference, and phase measurement vary inversely with antenna separation, and the others remain constant.

Figures B-1 through B-4 show the rms azimuth error in seconds for various antenna azimuths and pass elevations. From these graphs it can be seen that the key to system improvement lies in decreasing the antenna separation measurement error to 0.1 cm or less. This accuracy requires laser ranging but is certainly possible.

A listing of the computer simulation used to generate the data in Tables B-1 and B-2 is given in Fig. B-5.

TABLE B-1

RESULTS OF PASSES WITH 100 METER BASELINE

SOUTHEAST TO NORTHEAST PASS ELEVATION AT CA IS 10.5 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	R	RMS TOTAL
0	2.2E+00	7.5E-01	1.4E-02	1.8E-01	5.1E-01	6.1E-01	3.4E+00	2.2E+00	1.3E+01	1.4E+01
10	2.0E+00	6.4E-01	2.7E-02	2.0E-01	4.9E-01	5.5E-01	1.3E+01	2.3E+00	1.3E+01	1.9E+01
20	1.7E+00	5.9E-01	3.7E-02	1.8E-01	4.4E-01	4.5E-01	2.1E+01	2.4E+00	1.3E+01	2.5E+01
30	1.4E+00	5.2E-01	4.5E-02	1.2E-01	3.6E-01	3.3E-01	2.7E+01	2.3E+00	1.2E+01	3.0E+01
40	9.2E-01	4.5E-01	4.9E-02	5.1E-02	2.8E-01	1.9E-01	3.0E+01	2.3E+00	1.2E+01	3.3E+01
50	4.7E-01	4.0E-01	5.1E-02	3.3E-02	1.9E-01	5.5E-02	3.1E+01	2.2E+00	1.2E+01	3.3E+01
60	5.0E-02	3.7E-01	4.9E-02	1.2E-01	1.0E-01	6.6E-02	2.9E+01	2.1E+00	1.2E+01	3.1E+01
70	3.2E-01	3.5E-01	4.4E-02	2.1E-01	2.5E-02	1.7E-01	2.5E+01	1.9E+00	1.2E+01	2.8E+01
80	6.1E-01	3.6E-01	3.7E-02	2.9E-01	3.7E-02	2.4E-01	1.9E+01	1.7E+00	1.2E+01	2.3E+01
90	7.9E-01	3.9E-01	2.8E-02	3.5E-01	8.0E-02	2.8E-01	1.2E+01	1.5E+00	1.2E+01	1.7E+01

SOUTHEAST TO NORTHEAST PASS ELEVATION AT CA IS 20.3 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	R	RMS TOTAL
0	2.3E+00	9.9E-01	1.1E-02	4.0E-02	5.4E-01	6.2E-01	5.0E+00	3.7E+00	1.2E+01	1.3E+01
10	2.4E+00	1.0E+00	4.2E-02	7.4E-02	6.1E-01	6.1E-01	5.8E+00	4.3E+00	1.2E+01	1.4E+01
20	2.3E+00	1.0E+00	7.4E-02	7.7E-02	6.1E-01	5.2E-01	1.7E+01	4.7E+00	1.2E+01	2.1E+01
30	1.8E+00	8.8E-01	9.8E-02	4.5E-02	5.5E-01	3.6E-01	2.5E+01	4.8E+00	1.2E+01	2.8E+01
40	1.2E+00	7.1E-01	1.1E-01	1.4E-02	4.2E-01	1.6E-01	2.9E+01	4.5E+00	1.2E+01	3.2E+01
50	4.8E-01	5.2E-01	1.1E-01	8.5E-02	2.7E-01	3.4E-02	2.8E+01	4.0E+00	1.2E+01	3.1E+01
60	9.6E-02	3.6E-01	9.2E-02	1.5E-01	1.3E-01	1.8E-01	2.3E+01	3.3E+00	1.1E+01	2.6E+01
70	5.1E-01	2.5E-01	7.2E-02	2.1E-01	1.5E-02	2.7E-01	1.6E+01	2.7E+00	1.1E+01	2.0E+01
80	7.5E-01	1.8E-01	4.9E-02	2.5E-01	6.5E-02	3.0E-01	8.4E+00	2.0E+00	1.0E+01	1.3E+01
90	8.2E-01	1.5E-01	2.6E-02	2.7E-01	1.1E-01	2.9E-01	6.2E-01	1.4E+00	9.9E+00	1.0E+01

SOUTHEAST TO NORTHEAST PASS ELEVATION AT CA IS 39.1 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	R	RMS TOTAL
0	2.7E+00	1.1E+00	1.4E-02	5.8E-02	5.9E-01	7.3E-01	1.2E+01	6.5E+00	1.2E+01	1.8E+01
10	3.1E+00	1.4E+00	5.6E-02	4.5E-02	7.9E-01	7.5E-01	1.9E+00	7.9E+00	1.3E+01	1.6E+01
20	2.9E+00	1.5E+00	1.3E-01	7.2E-02	8.6E-01	6.0E-01	1.7E+01	8.4E+00	1.3E+01	2.4E+01
30	2.1E+00	1.3E+00	1.7E-01	1.3E-01	7.4E-01	3.2E-01	2.7E+01	7.5E+00	1.3E+01	3.1E+01
40	1.1E+00	9.6E-01	1.7E-01	1.9E-01	5.3E-01	5.0E-02	2.9E+01	5.9E+00	1.2E+01	3.1E+01
50	3.3E-01	6.6E-01	1.4E-01	2.3E-01	3.2E-01	1.3E-01	2.4E+01	4.3E+00	1.0E+01	2.7E+01
60	1.8E-01	4.3E-01	1.0E-01	2.6E-01	1.5E-01	2.2E-01	1.8E+01	3.0E+00	9.3E+00	2.0E+01
70	4.7E-01	2.8E-01	6.8E-02	2.7E-01	3.5E-02	2.5E-01	1.1E+01	2.1E+00	8.5E+00	1.4E+01
80	6.1E-01	1.8E-01	3.6E-02	2.7E-01	4.2E-02	2.4E-01	4.8E+00	1.4E+00	7.9E+00	9.4E+00
90	6.4E-01	1.2E-01	9.1E-03	2.6E-01	9.0E-02	2.1E-01	7.9E-01	8.0E-01	7.5E+00	7.6E+00

TABLE B-1 (cont'd)

RESULTS OF PASSES WITH 100 METER BASELINE

SOUTHEAST TO NORTHEAST PASS ELEVATION AT CA IS 63.6 DEG.

ANT	ERROR IN SECONDS PER VARIABLE									RMS
AZ	LAT	LON	HT	XS	YS	ZS	D	DH	R	TOTAL
0	3.2E+00	1.1E+00	7.7E-02	1.3E-01	5.9E-01	9.2E-01	2.0E+01	9.3E+00	1.3E+01	2.6E+01
10	4.2E+00	1.9E+00	4.0E-02	2.2E-01	1.0E+00	1.0E+00	3.9E+00	1.3E+01	1.5E+01	2.1E+01
20	4.0E+00	2.2E+00	1.8E-01	3.2E-01	1.2E+00	7.8E-01	1.8E+01	1.3E+01	1.6E+01	2.8E+01
30	2.6E+00	1.9E+00	2.4E-01	3.7E-01	1.0E+00	3.3E-01	2.9E+01	1.1E+01	1.4E+01	3.5E+01
40	1.2E+00	1.3E+00	2.2E-01	3.6E-01	6.6E-01	1.8E-03	2.8E+01	7.1E+00	1.2E+01	3.2E+01
50	3.4E-01	7.4E-01	1.6E-01	3.2E-01	3.8E-01	1.6E-01	2.2E+01	4.5E+00	1.0E+01	2.4E+01
60	1.5E-01	4.0E-01	1.1E-01	2.8E-01	1.8E-01	2.1E-01	1.5E+01	2.8E+00	9.0E+00	1.7E+01
70	3.9E-01	1.7E-01	6.5E-02	2.5E-01	5.0E-02	2.2E-01	8.4E+00	1.8E+00	8.0E+00	1.2E+01
80	4.9E-01	2.3E-02	2.8E-02	2.2E-01	3.6E-02	1.9E-01	3.1E+00	1.0E+00	7.4E+00	8.1E+00
90	5.1E-01	7.6E-02	3.5E-03	1.9E-01	9.3E-02	1.5E-01	1.6E+00	4.6E-01	7.0E+00	7.2E+00

SOUTHEAST TO NORTHEAST PASS ELEVATION AT CA IS 87.0 DEG.

ANT	ERROR IN SECONDS PER VARIABLE									RMS
AZ	LAT	LON	HT	XS	YS	ZS	D	DH	R	TOTAL
0	3.7E+00	7.6E-01	2.2E-01	1.2E-01	4.3E-01	1.2E+00	3.7E+01	1.2E+01	1.7E+01	4.3E+01
10	6.6E+00	2.7E+00	9.4E-02	5.1E-01	1.3E+00	1.8E+00	2.2E+01	2.2E+01	2.3E+01	3.9E+01
20	6.8E+00	3.7E+00	2.3E-01	8.2E-01	2.0E+00	1.4E+00	2.4E+01	2.4E+01	2.4E+01	4.3E+01
30	3.2E+00	2.0E+00	3.0E-01	5.5E-01	1.3E+00	4.0E-01	3.9E+01	1.3E+01	1.9E+01	4.5E+01
40	1.2E+00	6.9E-01	2.2E-01	2.9E-01	6.5E-01	8.4E-04	2.9E+01	6.3E+00	1.4E+01	3.3E+01
50	3.7E-01	3.8E-02	1.5E-01	1.4E-01	3.3E-01	1.1E-01	2.0E+01	3.3E+00	1.1E+01	2.3E+01
60	1.5E-02	3.0E-01	9.1E-02	5.9E-02	1.5E-01	1.3E-01	1.2E+01	1.8E+00	9.0E+00	1.5E+01
70	1.4E-01	5.0E-01	5.0E-02	7.3E-03	4.0E-02	1.1E-01	6.6E+00	9.4E-01	7.8E+00	1.0E+01
80	2.0E-01	6.2E-01	1.8E-02	2.8E-02	3.2E-02	8.8E-02	2.1E+00	4.2E-01	7.1E+00	7.4E+00
90	2.1E-01	7.0E-01	9.3E-03	5.4E-02	8.3E-02	5.5E-02	1.8E+00	5.1E-02	6.6E+00	6.9E+00

SOOTHEAST TO NORTHWEST PASS ELEVATION AT CA IS 89.6 DEG.

ANT	ERROR IN SECONDS PER VARIABLE									RMS
AZ	LAT	LON	HT	XS	YS	ZS	D	DR	R	TOTAL
0	3.8E+00	3.4E-01	3.1E-01	3.2E-02	2.9E-01	1.3E+00	4.9E+01	1.3E+01	2.0E+01	5.5E+01
10	8.6E+00	3.0E+00	2.2E-01	6.1E-01	1.6E+00	2.4E+00	3.6E+01	2.9E+01	3.0E+01	5.6E+01
20	8.2E+00	4.0E+00	2.8E-01	1.0E+00	2.4E+00	1.7E+00	4.0E+01	2.9E+01	3.1E+01	5.9E+01
30	2.9E+00	1.5E+00	3.0E-01	5.0E-01	1.2E+00	3.4E-01	4.5E+01	1.1E+01	2.0E+01	5.1E+01
40	1.1E+00	3.5E-01	2.0E-01	2.2E-01	5.9E-01	1.0E-03	3.1E+01	4.8E+00	1.4E+01	3.4E+01
50	4.0E-01	1.7E-01	1.3E-01	9.0E-02	3.1E-01	7.2E-02	2.0E+01	2.4E+00	1.1E+01	2.3E+01
60	1.3E-01	4.3E-01	8.3E-02	1.8E-02	1.6E-01	7.8E-02	1.2E+01	1.2E+00	8.9E+00	1.5E+01
70	4.4E-03	5.8E-01	4.6E-02	2.7E-02	6.5E-02	6.2E-02	7.0E+00	6.3E-01	7.7E+00	1.0E+01
80	4.6E-02	6.8E-01	1.8E-02	5.9E-02	1.5E-03	3.7E-02	2.6E+00	2.5E-01	7.0E+00	7.5E+00
90	5.8E-02	7.6E-01	7.2E-03	8.2E-02	4.6E-02	9.0E-03	1.2E+00	2.1E-02	6.6E+00	6.7E+00

TABLE B-1 (cont'd)
RESULTS OF PASSES WITH 100 METER BASELINE

SOUTHEAST TO NORTHWEST PASS ELEVATION AT CA IS 88.5 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	R	RMS TOTAL
0	3.5E+00	3.2E-01	4.3E-01	1.2E-01	1.5E-02	1.4E+00	6.6E+01	1.2E+01	2.4E+01	7.1E+01
10	1.2E+01	3.2E+00	5.0E-01	6.6E-01	1.9E+00	3.7E+00	6.9E+01	4.1E+01	4.5E+01	9.3E+01
20	9.0E+00	3.8E+00	4.1E-01	1.1E+00	2.8E+00	1.7E+00	7.6E+01	3.0E+01	4.0E+01	9.1E+01
30	2.3E+00	8.1E-01	3.1E-01	3.5E-01	1.1E+00	1.9E-01	5.2E+01	7.9E+00	2.1E+01	5.7E+01
40	8.7E-01	3.2E-02	1.9E-01	1.2E-01	5.2E-01	1.7E-02	3.2E+01	3.0E+00	1.4E+01	3.5E+01
50	4.2E-01	3.7E-01	1.2E-01	2.9E-02	3.0E-01	4.3E-02	2.0E+01	1.4E+00	1.1E+01	2.3E+01
60	2.4E-01	5.4E-01	7.8E-02	2.3E-02	1.8E-01	3.2E-02	1.3E+01	7.2E-01	8.8E+00	1.6E+01
70	1.5E-01	6.5E-01	4.5E-02	5.7E-02	9.7E-02	1.2E-02	7.5E+00	3.5E-01	7.7E+00	1.1E+01
80	1.2E-01	7.2E-01	1.9E-02	8.2E-02	4.1E-02	1.1E-02	3.2E+00	1.2E-01	7.0E+00	7.7E+00
90	1.0E-01	7.8E-01	3.8E-03	1.0E-01	3.2E-03	3.7E-02	5.5E-01	5.1E-02	6.6E+00	6.6E+00

SOUTHWEST TO NORTHWEST PASS ELEVATION AT CA IS 83.8 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	R	RMS TOTAL
0	2.2E+00	1.2E+00	5.3E-01	3.3E-01	4.8E-01	1.2E+00	8.5E+01	6.7E+00	2.8E+01	9.0E+01
10	1.9E+01	2.0E+00	1.4E+00	2.0E-01	1.8E+00	6.6E+00	1.9E+02	6.2E+01	8.7E+01	2.1E+02
20	5.9E+00	1.9E+00	6.4E-01	6.5E-01	2.4E+00	7.2E-01	1.2E+02	1.7E+01	4.8E+01	1.3E+02
30	1.3E+00	7.4E-02	3.1E-01	1.6E-01	8.4E-01	3.7E-02	5.7E+01	3.2E+00	2.2E+01	6.1E+01
40	6.5E-01	3.4E-01	1.9E-01	3.5E-02	4.6E-01	5.3E-02	3.3E+01	1.2E+00	1.4E+01	3.6E+01
50	4.3E-01	5.1E-01	1.2E-01	2.1E-02	2.9E-01	2.5E-02	2.1E+01	5.4E-01	1.1E+01	2.4E+01
60	3.4E-01	6.1E-01	7.8E-02	5.4E-02	2.0E-01	4.8E-03	1.4E+01	2.6E-01	8.8E+00	1.6E+01
70	2.9E-01	6.8E-01	4.6E-02	7.7E-02	1.3E-01	3.1E-02	8.2E+00	1.2E-01	7.7E+00	1.1E+01
80	2.7E-01	7.3E-01	2.1E-02	9.4E-02	8.1E-02	5.6E-02	3.9E+00	2.8E-02	7.0E+00	8.0E+00
90	2.5E-01	7.7E-01	6.9E-04	1.1E-01	3.9E-02	8.0E-02	1.1E-01	4.0E-02	6.6E+00	6.7E+00

SOUTHWEST TO NORTHWEST PASS ELEVATION AT CA IS 58.0 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	R	RMS TOTAL
0	3.8E+00	7.7E-01	2.5E-01	2.9E-01	1.2E+00	6.1E-01	5.9E+01	1.2E+01	2.5E+01	6.5E+01
10	9.4E+00	7.2E-01	2.5E-02	4.1E-01	2.0E+00	2.4E+00	1.7E+01	3.0E+01	4.0E+01	5.4E+01
20	5.1E+00	3.4E-02	4.5E-01	5.9E-02	3.1E-01	1.8E+00	7.4E+01	1.8E+01	3.3E+01	8.3E+01
30	1.0E+00	6.1E-02	3.0E-01	1.7E-01	3.3E-01	6.0E-01	5.4E+01	6.0E+00	2.0E+01	5.8E+01
40	3.7E-02	2.2E-02	1.9E-01	1.6E-01	3.7E-01	1.9E-01	3.5E+01	2.4E+00	1.4E+01	3.8E+01
50	3.9E-01	8.6E-02	1.2E-01	1.4E-01	3.4E-01	1.8E-02	2.3E+01	1.1E+00	1.1E+01	2.6E+01
60	5.2E-01	1.3E-01	7.8E-02	1.2E-01	2.9E-01	6.8E-02	1.5E+01	4.9E-01	9.2E+00	1.8E+01
70	5.7E-01	1.7E-01	4.6E-02	1.1E-01	2.5E-01	1.2E-01	9.2E+00	1.6E-01	8.1E+00	1.2E+01
80	5.7E-01	2.0E-01	2.2E-02	9.2E-02	2.0E-01	1.5E-01	4.3E+00	6.0E-02	7.4E+00	8.6E+00
90	5.6E-01	2.3E-01	3.5E-04	7.9E-02	1.6E-01	1.7E-01	1.8E-01	2.2E-01	7.0E+00	7.0E+00

TABLE B-1 (cont'd)

RESULTS OF PASSES WITH 100 METER BASELINE

SOUTHWEST TO NORTHWEST PASS ELEVATION AT CA IS 25.5 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	N	RMS TOTAL
0	3.2E+00	5.1E-01	5.3E-02	6.9E-03	8.1E-01	7.9E-01	1.3E+01	7.3E+00	1.6E+01	2.2E+01
10	3.6E+00	7.0E-01	8.5E-03	4.1E-02	8.0E-01	9.9E-01	6.0E+00	8.0E+00	1.8E+01	2.1E+01
20	3.1E+00	7.3E-01	7.2E-02	1.1E-01	5.6E-01	9.3E-01	2.6E+01	7.1E+00	1.8E+01	3.2E+01
30	1.9E+00	5.8E-01	1.0E-01	1.6E-01	2.3E-01	5.4E-01	3.5E+01	5.2E+00	1.6E+01	3.9E+01
40	7.7E-01	3.8E-01	9.9E-02	1.7E-01	3.7E-02	3.2E-01	3.5E+01	3.2E+00	1.5E+01	3.8E+01
50	2.7E-02	2.0E-01	7.8E-02	1.7E-01	1.9E-01	6.9E-02	2.8E+01	1.7E+00	1.3E+01	3.1E+01
60	5.0E-01	7.4E-02	5.3E-02	1.5E-01	2.6E-01	9.5E-02	2.1E+01	7.2E-01	1.1E+01	2.4E+01
70	7.4E-01	1.3E-02	2.8E-02	1.4E-01	2.7E-01	2.0E-01	1.3E+01	4.7E-02	1.0E+01	1.7E+01
80	8.4E-01	7.1E-02	6.1E-03	1.2E-01	2.5E-01	2.5E-01	6.3E+00	4.4E-01	9.7E+00	1.2E+01
90	8.4E-01	1.1E-01	1.4E-02	1.0E-01	2.2E-01	2.8E-01	2.1E-01	8.3E-01	9.2E+00	9.3E+00

SOUTHWEST TO NORTHWEST PASS ELEVATION AT CA IS 15.4 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	N	RMS TOTAL
0	2.4E+00	5.8E-01	2.8E-02	3.9E-03	6.1E-01	6.4E-01	9.2E+00	3.9E+00	1.3E+01	1.7E+01
10	2.6E+00	7.1E-01	3.8E-03	2.8E-02	6.2E-01	7.4E-01	3.9E+00	4.0E+00	1.4E+01	1.6E+01
20	2.5E+00	7.7E-01	2.2E-02	8.0E-02	5.3E-01	7.3E-01	1.8E+01	3.8E+00	1.5E+01	2.4E+01
30	1.8E+00	7.2E-01	4.1E-02	1.4E-01	3.4E-01	5.7E-01	3.0E+01	3.1E+00	1.5E+01	3.3E+01
40	9.8E-01	5.8E-01	4.6E-02	1.8E-01	1.1E-01	3.3E-01	3.3E+01	2.1E+00	1.4E+01	3.6E+01
50	1.8E-01	4.1E-01	3.9E-02	2.0E-01	7.4E-02	9.3E-02	3.0E+01	1.1E+00	1.4E+01	3.3E+01
60	3.9E-01	2.5E-01	2.6E-02	2.0E-01	1.9E-01	9.0E-02	2.3E+01	2.9E-01	1.2E+01	2.6E+01
70	7.1E-01	1.4E-01	9.5E-03	1.9E-01	2.4E-01	2.1E-01	1.4E+01	3.0E-01	1.2E+01	1.8E+01
80	8.4E-01	5.8E-02	5.8E-03	1.7E-01	2.5E-01	2.7E-01	5.6E+00	7.4E-01	1.1E+01	1.2E+01
90	8.3E-01	1.2E-02	1.9E-02	1.4E-01	2.2E-01	2.8E-01	2.1E+00	1.1E+00	1.0E+01	1.1E+01

SOUTHWEST TO NORTHWEST PASS ELEVATION AT CA IS 10.5 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	N	RMS TOTAL
0	2.0E+00	4.2E-01	1.9E-02	6.4E-02	5.4E-01	5.6E-01	1.2E+01	2.6E+00	1.3E+01	1.8E+01
10	2.4E+00	5.2E-01	5.8E-03	5.6E-02	6.0E-01	6.8E-01	2.4E+00	2.8E+00	1.4E+01	1.5E+01
20	2.5E+00	6.4E-01	1.1E-02	1.2E-02	6.0E-01	7.5E-01	1.0E+01	3.0E+00	1.6E+01	1.9E+01
30	2.3E+00	7.5E-01	2.9E-02	8.1E-02	4.9E-01	7.0E-01	2.4E+01	2.9E+00	1.7E+01	3.0E+01
40	1.5E+00	7.9E-01	3.9E-02	2.1E-01	2.5E-01	4.8E-01	3.3E+01	2.2E+00	1.8E+01	3.8E+01
50	4.4E-01	7.2E-01	3.5E-02	3.2E-01	2.9E-02	1.7E-01	3.1E+01	1.2E+00	1.8E+01	3.6E+01
60	4.1E-01	5.9E-01	2.0E-02	3.6E-01	2.2E-01	1.0E-01	2.1E+01	1.8E-01	1.6E+01	2.6E+01
70	8.2E-01	4.6E-01	3.2E-03	3.4E-01	3.0E-01	2.5E-01	7.9E+00	5.2E-01	1.5E+01	1.7E+01
80	9.1E-01	3.6E-01	1.1E-02	3.0E-01	2.9E-01	2.9E-01	2.9E+00	9.4E-01	1.3E+01	1.4E+01
90	8.1E-01	2.9E-01	2.2E-02	2.5E-01	2.3E-01	2.8E-01	1.1E+01	1.2E+00	1.2E+01	1.6E+01

TABLE B 2

RESULTS OF PASSES WITH 50 METER BASELINE

SOUTHEAST TO NORTHEAST PASS ELEVATION AT CA IS 10.5 DEG.

ANT AZ	LAT	LON	HT	XS	YS	ZS	D	DH	M	RMS TOTAL
	ERROR IN SECONDS PER VARIABLE									
0	2.2E+00	7.5E-01	1.4E-02	1.8E-01	5.1E-01	6.1E-01	6.8E+00	4.4E+00	2.7E+01	2.8E+01
10	2.0E+00	6.8E-01	2.7E-02	2.0E-01	4.7E-01	5.5E-01	2.7E+01	4.7E+00	2.6E+01	3.7E+01
20	1.7E+00	5.9E-01	3.7E-02	1.8E-01	4.5E-01	4.5E-01	4.3E+01	4.7E+00	2.5E+01	5.0E+01
30	1.4E+00	5.2E-01	4.5E-02	1.2E-01	3.6E-01	3.3E-01	5.4E+01	4.7E+00	2.4E+01	6.0E+01
40	9.2E-01	4.5E-01	4.9E-02	5.1E-02	2.8E-01	1.9E-01	6.1E+01	4.6E+00	2.4E+01	6.5E+01
50	4.7E-01	4.0E-01	5.1E-02	3.3E-02	1.3E-01	5.5E-02	6.2E+01	4.4E+00	2.3E+01	6.6E+01
60	5.0E-02	3.7E-01	4.9E-02	1.2E-01	1.0E-01	6.6E-02	5.8E+01	4.1E+00	2.3E+01	6.3E+01
70	3.2E-01	3.5E-01	4.4E-02	2.1E-01	2.5E-02	1.7E-01	5.0E+01	3.8E+00	2.3E+01	5.6E+01
80	6.1E-01	3.6E-01	3.7E-02	2.9E-01	3.7E-02	2.4E-01	3.9E+01	3.5E+00	2.4E+01	4.5E+01
90	7.9E-01	3.9E-01	2.8E-02	3.5E-01	8.2E-02	2.8E-01	2.4E+01	3.0E+00	2.4E+01	3.4E+01

SOUTHEAST TO NORTHEAST PASS ELEVATION AT CA IS 20.3 DEG.

ANT AZ	LAT	LON	HT	XS	YS	ZS	D	DH	M	RMS TOTAL
	ERROR IN SECONDS PER VARIABLE									
0	2.3E+00	9.9E-01	1.1E-02	4.0E-02	5.4E-01	6.2E-01	9.9E+00	7.5E+00	2.3E+01	2.6E+01
10	2.4E+00	1.0E+00	4.2E-02	7.4E-02	6.1E-01	6.1E-01	1.2E+01	8.6E+00	2.4E+01	2.8E+01
20	2.3E+00	1.0E+00	7.4E-02	7.7E-02	6.1E-01	5.2E-01	3.3E+01	9.4E+00	2.5E+01	4.3E+01
30	1.8E+00	8.8E-01	9.8E-02	4.5E-02	5.5E-01	3.6E-01	5.0E+01	9.5E+00	2.5E+01	5.7E+01
40	1.2E+00	7.1E-01	1.1E-01	1.4E-02	4.2E-01	1.6E-01	5.8E+01	9.0E+00	2.4E+01	6.3E+01
50	4.8E-01	5.2E-01	1.1E-01	8.5E-02	2.7E-01	3.4E-02	5.6E+01	7.9E+00	2.3E+01	6.1E+01
60	9.6E-02	3.6E-01	9.2E-02	1.5E-01	1.6E-01	1.8E-01	4.6E+01	6.7E+00	2.2E+01	5.2E+01
70	5.1E-01	2.5E-01	7.2E-02	2.1E-01	1.5E-02	2.7E-01	3.3E+01	5.3E+00	2.1E+01	3.9E+01
80	7.5E-01	1.8E-01	4.9E-02	2.5E-01	6.5E-02	3.0E-01	1.7E+01	4.1E+00	2.0E+01	2.7E+01
90	8.2E-01	1.5E-01	2.6E-02	2.7E-01	1.1E-01	2.9E-01	1.2E+00	2.9E+00	2.0E+01	2.0E+01

SOUTHEAST TO NORTHEAST PASS ELEVATION AT CA IS 39.1 DEG.

ANT AZ	LAT	LON	HT	XS	YS	ZS	D	DH	M	RMS TOTAL
	ERROR IN SECONDS PER VARIABLE									
0	2.7E+00	1.1E+00	1.4E-02	5.8E-02	5.3E-01	7.3E-01	2.4E+01	1.3E+01	2.4E+01	3.6E+01
10	3.1E+00	1.4E+00	5.6E-02	4.5E-02	7.9E-01	7.5E-01	3.7E+00	1.6E+01	2.6E+01	3.1E+01
20	2.9E+00	1.5E+00	1.3E-01	7.2E-02	8.6E-01	6.0E-01	3.5E+01	1.7E+01	2.7E+01	4.7E+01
30	2.1E+00	1.3E+00	1.7E-01	1.3E-01	7.4E-01	3.2E-01	5.5E+01	1.5E+01	2.5E+01	6.2E+01
40	1.1E+00	9.6E-01	1.7E-01	1.9E-01	5.3E-01	5.0E-02	5.7E+01	1.2E+01	2.3E+01	6.3E+01
50	3.3E-01	6.6E-01	1.4E-01	2.3E-01	3.2E-01	1.3E-01	4.8E+01	8.7E+00	2.1E+01	5.3E+01
60	1.8E-01	4.3E-01	1.0E-01	2.6E-01	1.5E-01	2.2E-01	3.5E+01	6.1E+00	1.9E+01	4.0E+01
70	4.7E-01	2.8E-01	6.8E-02	2.7E-01	3.5E-02	2.5E-01	2.2E+01	4.2E+00	1.7E+01	2.8E+01
80	6.1E-01	1.8E-01	3.6E-02	2.7E-01	4.3E-02	2.4E-01	9.6E+00	2.7E+00	1.6E+01	1.9E+01
90	6.4E-01	1.2E-01	9.1E-03	2.6E-01	9.0E-02	2.1E-01	1.6E+00	1.6E+00	1.5E+01	1.5E+01

TABLE B-2 (cont'd)

RESULTS OF PASSES WITH 50 METER BASELINE

SOOTHEAST TO NORTHEAST PASS ELEVATION AT CA IS 63.6 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	H	RMS TOTAL
0	3.2E+00	1.1E+00	7.7E-02	1.3E-01	5.9E-01	9.2E-01	3.9E+01	1.9E+01	2.7E+01	5.1E+01
10	4.2E+00	1.9E+00	4.0E-02	2.2E-01	1.0E+00	1.0E+00	7.9E+00	2.5E+01	3.0E+01	4.1E+01
20	4.0E+00	2.2E+00	1.8E-01	3.2E-01	1.2E+00	7.8E-01	3.5E+01	2.7E+01	3.2E+01	5.5E+01
30	2.6E+00	1.9E+00	2.4E-01	3.7E-01	1.0E+00	[illegible].3E-01	5.9E+01	2.1E+01	2.9E+01	6.9E+01
40	1.2E+00	1.3E+00	2.2E-01	3.6E-01	6.6E-01	1.8E-03	5.6E+01	1.4E+01	2.5E+01	6.3E+01
50	3.4E-01	7.4E-01	1.6E-01	3.2E-01	3.8E-01	1.6E-01	4.3E+01	9.0E+00	2.1E+01	4.9E+01
60	1.5E-01	4.0E-01	1.1E-01	2.8E-01	1.8E-01	2.1E-01	2.9E+01	5.7E+00	1.8E+01	3.5E+01
70	3.9E-01	1.7E-01	6.5E-02	2.5E-01	5.0E-02	2.2E-01	1.7E+01	3.5E+00	1.6E+01	2.4E+01
80	4.9E-01	2.3E-02	2.8E-02	2.2E-01	3.6E-02	1.9E-01	6.2E+00	2.0E+00	1.5E+01	1.6E+01
90	5.1E-01	7.6E-02	3.5E-03	1.9E-01	9.3E-02	[illegible].5E-01	3.1E+00	9.2E-01	1.4E+01	1.4E+01

SOUTHEAST TO NORTHEAST PASS ELEVATION AT CA IS 87.0 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	H	RMS TOTAL
0	3.7E+00	7.6E-01	2.2E-01	1.2E-01	4.3E-01	1.2E+00	7.5E+01	2.4E+01	3.4E+01	8.6E+01
10	6.6E+00	2.7E+00	9.4E-02	5.1E-01	1.3E+00	1.8E+00	4.5E+01	4.4E+01	4.6E+01	7.8E+01
20	6.8E+00	3.7E+00	2.3E-01	8.2E-01	2.0E+00	1.4E+00	4.9E+01	4.8E+01	4.9E+01	8.5E+01
30	3.2E+00	2.0E+00	3.0E-01	5.5E-01	1.3E+00	4.0E-01	7.7E+01	2.6E+01	3.8E+01	9.0E+01
40	1.2E+00	6.9E-01	2.2E-01	2.9E-01	6.5E-01	8.5E-04	5.9E+01	1.3E+01	2.8E+01	6.6E+01
50	3.7E-01	3.8E-02	1.5E-01	1.4E-01	3.3E-01	1.1E-01	3.9E+01	6.5E+00	2.2E+01	4.5E+01
60	1.5E-02	3.0E-01	9.1E-02	5.9E-02	1.5E-01	1.3E-01	2.5E+01	3.5E+00	1.8E+01	3.1E+01
70	1.4E-01	5.0E-01	5.0E-02	7.3E-03	4.0E-02	1.1E-01	1.3E+01	1.9E+00	1.6E+01	2.1E+01
80	2.0E-01	6.2E-01	1.8E-02	2.8E-02	3.2E-02	8.8E-02	4.1E+00	8.4E-01	1.4E+01	1.5E+01
90	2.1E-01	7.0E-01	9.3E-03	5.4E-02	8.3E-02	5.5E-02	3.7E+00	1.0E-01	1.3E+01	1.4E+01

SOUTHEAST TO NORTHWEST PASS ELEVATION AT CA IS 89.6 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	H	RMS TOTAL
0	3.8E+00	3.4E-01	3.1E-01	3.2E-02	2.9E-01	1.3E+00	9.8E+01	2.5E+01	3.9E+01	1.1E+02
10	8.6E+00	3.0E+00	2.2E-01	6.1E-01	1.6E+00	2.4E+00	7.2E+01	5.8E+01	6.0E+01	1.1E+02
20	8.2E+00	4.0E+00	2.8E-01	1.0E+00	2.4E+00	1.7E+00	8.1E+01	5.7E+01	6.2E+01	1.2E+02
30	2.9E+00	1.5E+00	3.0E-01	5.0E-01	1.2E+00	3.4E-01	9.0E+01	2.3E+01	4.1E+01	1.0E+02
40	1.1E+00	3.5E-01	2.0E-01	2.2E-01	5.9E-01	1.0E-03	6.1E+01	9.7E+00	2.8E+01	6.8E+01
50	4.0E-01	1.7E-01	1.3E-01	9.0E-02	3.1E-01	7.2E-02	4.0E+01	4.7E+00	2.2E+01	4.5E+01
60	1.3E-01	4.3E-01	8.3E-02	1.8E-02	1.6E-01	7.8E-02	2.5E+01	2.5E+00	1.8E+01	3.1E+01
70	4.4E-03	5.8E-01	4.6E-02	2.7E-02	6.5E-02	6.2E-02	1.4E+01	1.3E+00	1.5E+01	2.1E+01
80	4.6E-02	6.8E-01	1.8E-02	5.9E-02	1.5E-03	3.7E-02	5.1E+00	5.0E-01	1.4E+01	1.5E+01
90	5.8E-02	7.6E-01	7.2E-03	8.2E-02	4.6E-02	9.0E-03	2.4E+00	4.1E-02	1.3E+01	1.3E+01

TABLE B-2 (cont'd)
RESULTS OF PASSES WITH 50 METER BASELINE

SOUTHEAST TO NORTHWEST PASS ELEVATION AT CA IS 88.5 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	M	RMS TOTAL
0	3.5E+00	3.2E-01	4.3E-01	1.2E-01	1.5E-02	1.4E+00	1.3E+02	2.3E+01	4.7E+01	1.4E+02
10	1.2E+01	3.2E+00	5.0E-01	6.6E-01	1.9E+00	3.7E+00	1.4E+02	8.3E+01	9.1E+01	1.8E+02
20	9.0E+00	3.8E+00	4.1E-01	1.1E+00	2.8E+00	1.7E+00	1.5E+02	6.1E+01	8.0E+01	1.8E+02
30	2.3E+00	8.1E-01	3.1E-01	3.5E-01	1.1E+00	1.9E-01	1.0E+02	1.6E+01	4.3E+01	1.1E+02
40	8.7E-01	3.2E-02	1.9E-01	1.2E-01	5.2E-01	1.7E-02	6.4E+01	6.0E+00	2.8E+01	7.0E+01
50	4.2E-01	3.7E-02	1.2E-01	2.9E-02	3.0E-01	4.3E-02	4.1E+01	2.8E+00	2.1E+01	4.6E+01
60	2.4E-01	5.4E-01	7.8E-02	2.3E-02	1.8E-01	3.2E-02	2.6E+01	1.4E+00	1.8E+01	3.1E+01
70	1.5E-01	6.5E-01	4.5E-02	5.7E-02	9.7E-02	1.2E-02	1.5E+01	7.0E-01	1.5E+01	2.1E+01
80	1.2E-01	7.2E-01	1.9E-02	8.2E-02	4.1E-02	1.1E-02	6.4E+00	2.3E-01	1.4E+01	1.5E+01
90	1.0E-01	7.8E-01	3.8E-03	1.0E-01	3.2E-03	3.7E-02	1.1E+00	1.0E-01	1.3E+01	1.3E+01

SOUTHWEST TO NORTHWEST PASS ELEVATION AT CA IS 83.8 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	M	RMS TOTAL
0	2.2E+00	1.2E+00	5.3E-01	3.3E-01	4.8E-01	1.2E+00	1.7E+02	1.3E+01	5.6E+01	1.8E+02
10	1.9E+01	2.0E+00	1.4E+00	2.0E-01	1.8E+00	6.6E+00	3.7E+02	1.2E+02	1.7E+02	4.3E+02
20	5.9E+00	1.9E+00	6.4E-01	6.5E-01	2.4E+00	7.2E-01	2.5E+02	3.5E+01	9.5E+01	2.7E+02
30	1.3E+00	7.4E-02	3.1E-01	1.6E-01	8.4E-01	3.7E-02	1.1E+02	6.4E+00	4.3E+01	1.2E+02
40	6.5E-01	3.4E-01	1.9E-01	3.5E-02	4.6E-01	5.3E-02	6.6E+01	2.3E+00	2.8E+01	7.2E+01
50	4.3E-01	5.1E-01	1.2E-01	2.1E-02	2.9E-01	2.5E-02	4.2E+01	1.1E+00	2.1E+01	4.7E+01
60	3.4E-01	6.1E-01	7.8E-02	5.4E-02	2.0E-01	4.8E-03	2.7E+01	5.3E-01	1.8E+01	3.2E+01
70	2.9E-01	6.8E-01	4.6E-02	7.7E-02	1.3E-01	3.1E-02	1.6E+01	2.4E-01	1.5E+01	2.2E+01
80	2.7E-01	7.3E-01	2.1E-02	9.4E-02	8.1E-02	5.6E-02	7.7E+00	5.6E-02	1.4E+01	1.6E+01
90	2.5E-01	7.7E-01	6.9E-04	1.1E-01	3.9E-02	8.0E-02	2.3E-01	8.0E-02	1.3E+01	1.3E+01

SOUTHWEST TO NORTHWEST PASS ELEVATION AT CA IS 58.0 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	M	RMS TOTAL
0	3.8E+00	7.7E-01	2.5E-01	2.9E-01	1.2E+00	6.1E-01	1.2E+02	2.5E+01	5.0E+01	1.3E+02
10	9.4E+00	7.2E-01	2.5E-02	4.1E-01	2.0E+00	2.4E+00	3.4E+01	6.0E+01	8.1E+01	1.1E+02
20	5.1E+00	3.4E-02	4.5E-01	5.9E-02	3.1E-01	1.8E+00	1.5E+02	3.7E+01	6.6E+01	1.7E+02
30	1.0E+00	6.1E-02	3.0E-01	1.7E-01	3.3E-01	6.0E-01	1.1E+02	1.2E+01	4.1E+01	1.2E+02
40	3.7E-02	2.2E-02	1.9E-01	1.6E-01	3.7E-01	1.9E-01	7.0E+01	4.8E+00	2.8E+01	7.6E+01
50	3.9E-01	8.6E-02	1.2E-01	1.4E-01	3.4E-01	1.8E-02	4.6E+01	2.2E+00	2.2E+01	5.1E+01
60	5.2E-01	1.3E-01	7.8E-02	1.2E-01	2.9E-01	6.8E-02	3.0E+01	9.8E-01	1.8E+01	3.5E+01
70	5.7E-01	1.7E-01	4.6E-02	1.1E-01	2.5E-01	1.2E-01	1.8E+01	3.1E-01	1.6E+01	2.4E+01
80	5.7E-01	2.0E-01	2.2E-02	9.2E-02	2.0E-01	1.5E-01	8.7E+00	1.2E-01	1.5E+01	1.7E+01
90	5.6E-01	2.3E-01	3.5E-04	7.9E-02	1.6E-01	1.7E-01	3.6E-01	4.4E-01	1.4E+01	1.4E+01

TABLE B-2 (cont'd)

RESULTS OF PASSES WITH 50 METER BASELINE

SOUTHWEST TO NORTHWEST PASS ELEVATION AT CA IS 25.5 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	M	RMS TOTAL
0	3.2E+00	5.1E-01	5.3E-02	6.9E-03	8.1E-01	7.9E-01	2.6E+01	1.5E+01	3.2E+01	4.4E+01
10	3.6E+00	7.0E-01	8.5E-03	4.1E-02	8.0E-01	9.9E-01	1.2E+01	1.6E+01	3.5E+01	4.1E+01
20	3.1E+00	7.3E-01	7.2E-02	1.1E-01	5.6E-01	9.3E-01	5.1E+01	1.4E+01	3.5E+01	6.4E+01
30	1.9E+00	5.8E-01	1.0E-01	1.6E-01	2.3E-01	6.4E-01	7.1E+01	1.0E+01	3.3E+01	7.9E+01
40	7.7E-01	3.8E-01	9.9E-02	1.7E-01	3.7E-02	3.2E-01	6.9E+01	6.4E+00	2.9E+01	7.5E+01
50	2.7E-02	2.0E-01	7.8E-02	1.7E-01	1.9E-01	6.9E-02	5.6E+01	3.4E+00	2.6E+01	6.2E+01
60	5.0E-01	7.4E-02	5.3E-02	1.5E-01	2.6E-01	9.5E-02	4.1E+01	1.4E+00	2.3E+01	4.7E+01
70	7.4E-01	1.3E-02	2.8E-02	1.4E-01	2.7E-01	2.0E-01	2.6E+01	9.5E-02	2.1E+01	3.3E+01
80	8.4E-01	7.1E-02	6.1E-03	1.2E-01	2.5E-01	2.5E-01	1.3E+01	8.8E-01	1.9E+01	2.3E+01
90	8.4E-01	1.1E-01	1.4E-02	1.0E-01	2.2E-01	2.8E-01	4.1E-01	1.7E+00	1.8E+01	1.9E+01

SOUTHWEST TO NORTHWEST PASS ELEVATION AT CA IS 15.4 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	M	RMS TOTAL
0	2.4E+00	5.8E-01	2.8E-02	3.9E-03	6.1E-01	6.4E-01	1.8E+01	7.8E+00	2.7E+01	3.4E+01
10	2.6E+00	7.1E-01	3.8E-03	2.8E-02	6.2E-01	7.4E-01	7.8E+00	8.0E+00	2.9E+01	3.1E+01
20	2.5E+00	7.7E-01	2.2E-02	8.0E-02	5.3E-01	7.3E-01	3.7E+01	7.5E+00	3.0E+01	4.8E+01
30	1.8E+00	7.2E-01	4.1E-02	1.4E-01	3.4E-01	5.7E-01	5.9E+01	6.1E+00	3.0E+01	6.7E+01
40	9.8E-01	5.8E-01	4.6E-02	1.8E-01	1.1E-01	3.3E-01	6.7E+01	4.1E+00	2.9E+01	7.3E+01
50	1.8E-01	4.1E-01	3.9E-02	2.0E-01	7.4E-02	9.3E-02	6.0E+01	2.2E+00	2.7E+01	6.6E+01
60	3.9E-01	2.5E-01	2.6E-02	2.0E-01	1.9E-01	9.0E-02	4.5E+01	5.8E-01	2.5E+01	5.2E+01
70	7.1E-01	1.4E-01	9.5E-03	1.9E-01	2.4E-01	2.1E-01	2.8E+01	6.0E-01	2.3E+01	3.6E+01
80	8.4E-01	5.8E-02	5.8E-03	1.7E-01	2.5E-01	2.7E-01	1.1E+01	1.5E+00	2.2E+01	2.4E+01
90	8.3E-01	1.2E-02	1.9E-02	1.4E-01	2.2E-01	2.8E-01	4.1E+00	2.1E+00	2.0E+01	2.1E+01

SOUTHWEST TO NORTHWEST PASS ELEVATION AT CA IS 10.5 DEG.

ANT AZ	ERROR IN SECONDS PER VARIABLE LAT	LON	HT	XS	YS	ZS	D	DH	M	RMS TOTAL
0	2.0E+00	4.2E-01	1.9E-02	6.4E-02	5.4E-01	5.6E-01	2.4E+01	5.1E+00	2.6E+01	3.5E+01
10	2.4E+00	5.2E-01	5.8E-03	5.6E-02	6.0E-01	6.8E-01	4.8E+00	5.7E+00	2.9E+01	3.0E+01
20	2.5E+00	6.4E-01	1.1E-02	1.2E-02	6.0E-01	7.5E-01	2.1E+01	6.0E+00	3.2E+01	3.9E+01
30	2.3E+00	7.5E-01	2.9E-02	8.1E-02	4.9E-01	7.0E-01	4.8E+01	5.7E+00	3.5E+01	6.0E+01
40	1.5E+00	7.9E-01	3.9E-02	2.1E-01	2.5E-01	4.8E-01	6.6E+01	4.4E+00	3.6E+01	7.6E+01
50	4.4E-01	7.2E-01	3.5E-02	3.2E-01	2.9E-02	1.7E-01	6.2E+01	2.3E+00	3.6E+01	7.2E+01
60	4.1E-01	5.9E-01	2.0E-02	3.6E-01	2.2E-01	1.0E-01	4.1E+01	3.5E-01	3.3E+01	5.3E+01
70	9.2E-01	4.6E-01	3.2E-03	3.4E-01	3.0E-01	2.5E-01	1.6E+01	1.0E+00	2.9E+01	3.3E+01
80	9.1E-01	3.6E-01	1.1E-02	3.0E-01	2.9E-01	2.9E-01	5.8E+00	1.9E+00	2.6E+01	2.7E+01
90	8.1E-01	2.9E-01	2.2E-02	2.5E-01	2.3E-01	2.8E-01	2.2E+01	2.4E+00	2.4E+01	3.3E+01

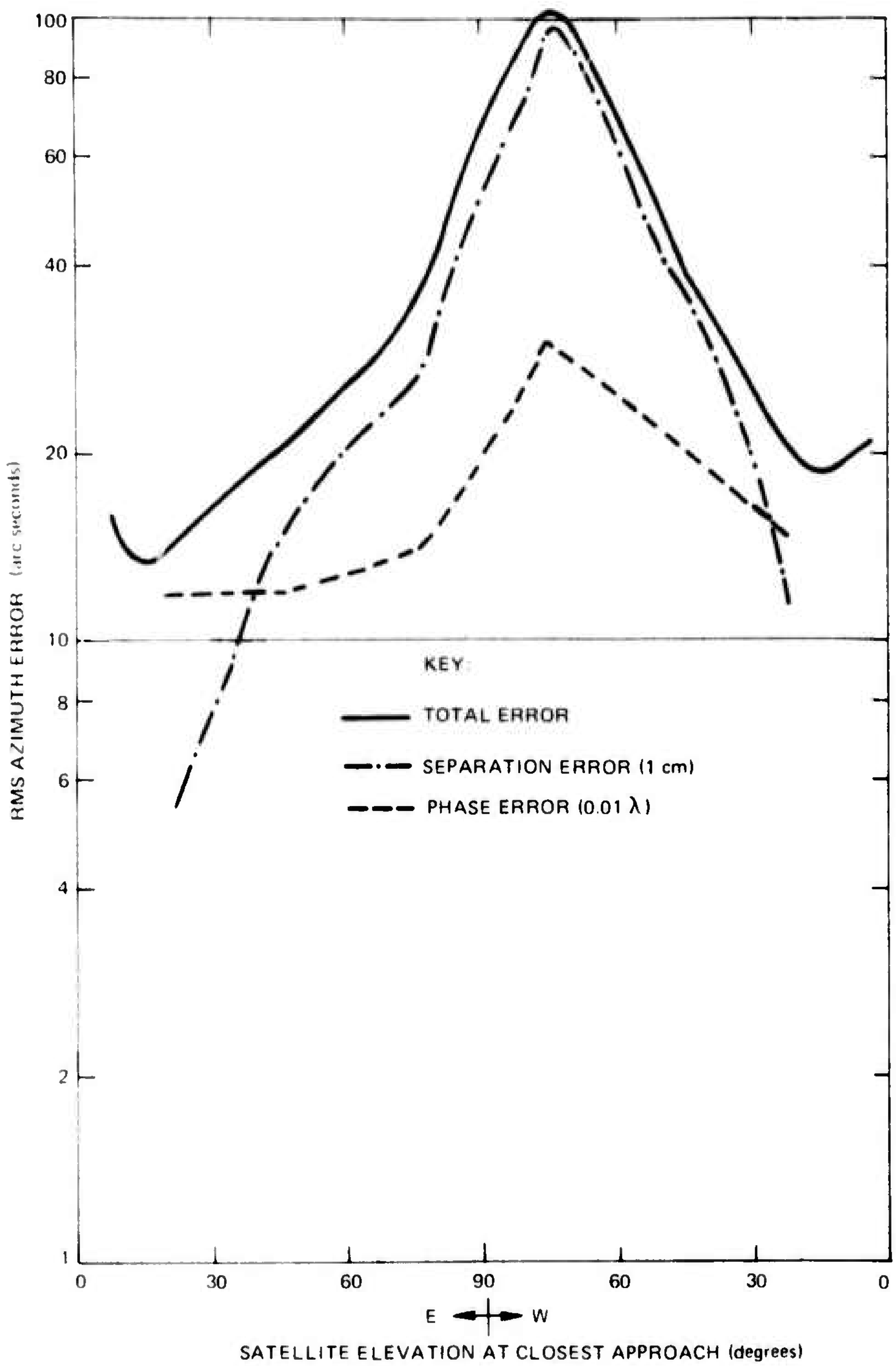

Fig. B-1 RMS AZIMUTH ERROR FOR SATELLITE PASSES OVER APL (AZ = 0°)

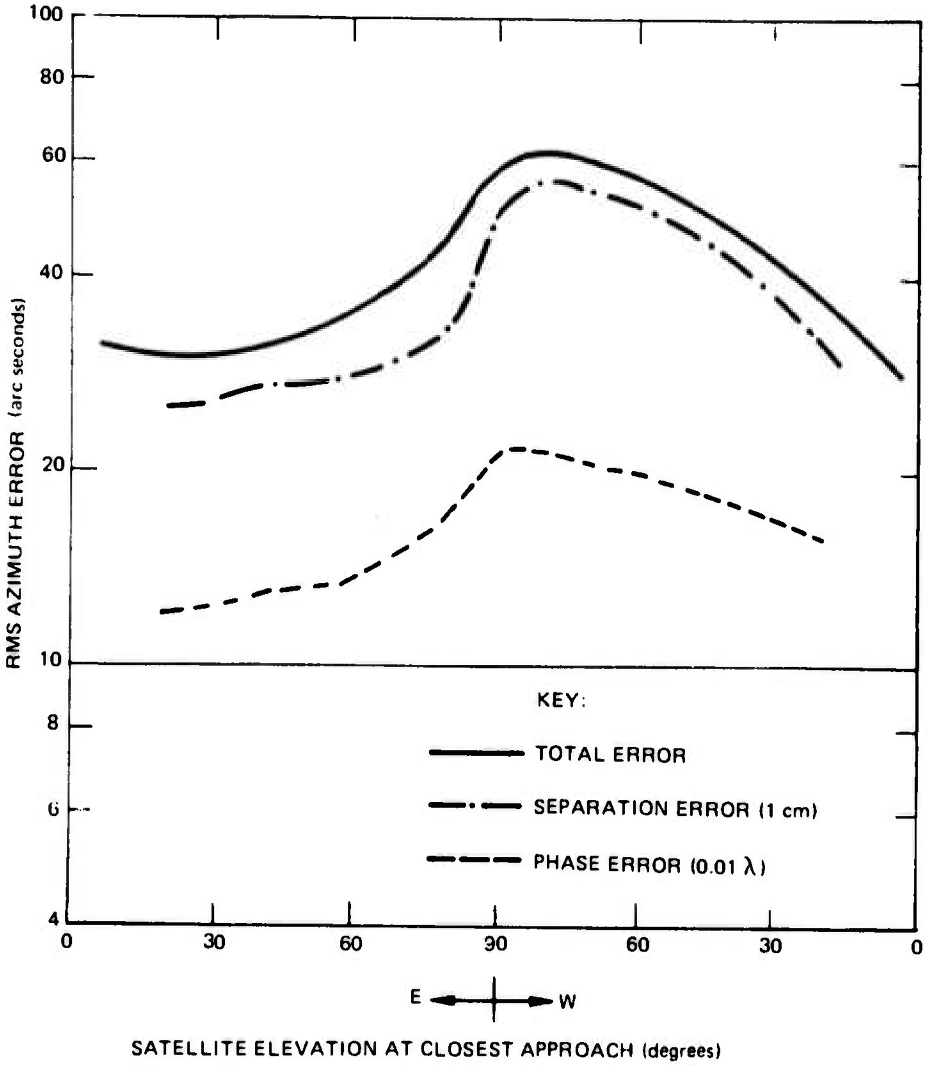

Fig. B-2 RMS AZIMUTH ERROR FOR SATELLITE PASSES OVER APL (AZ = 30°)

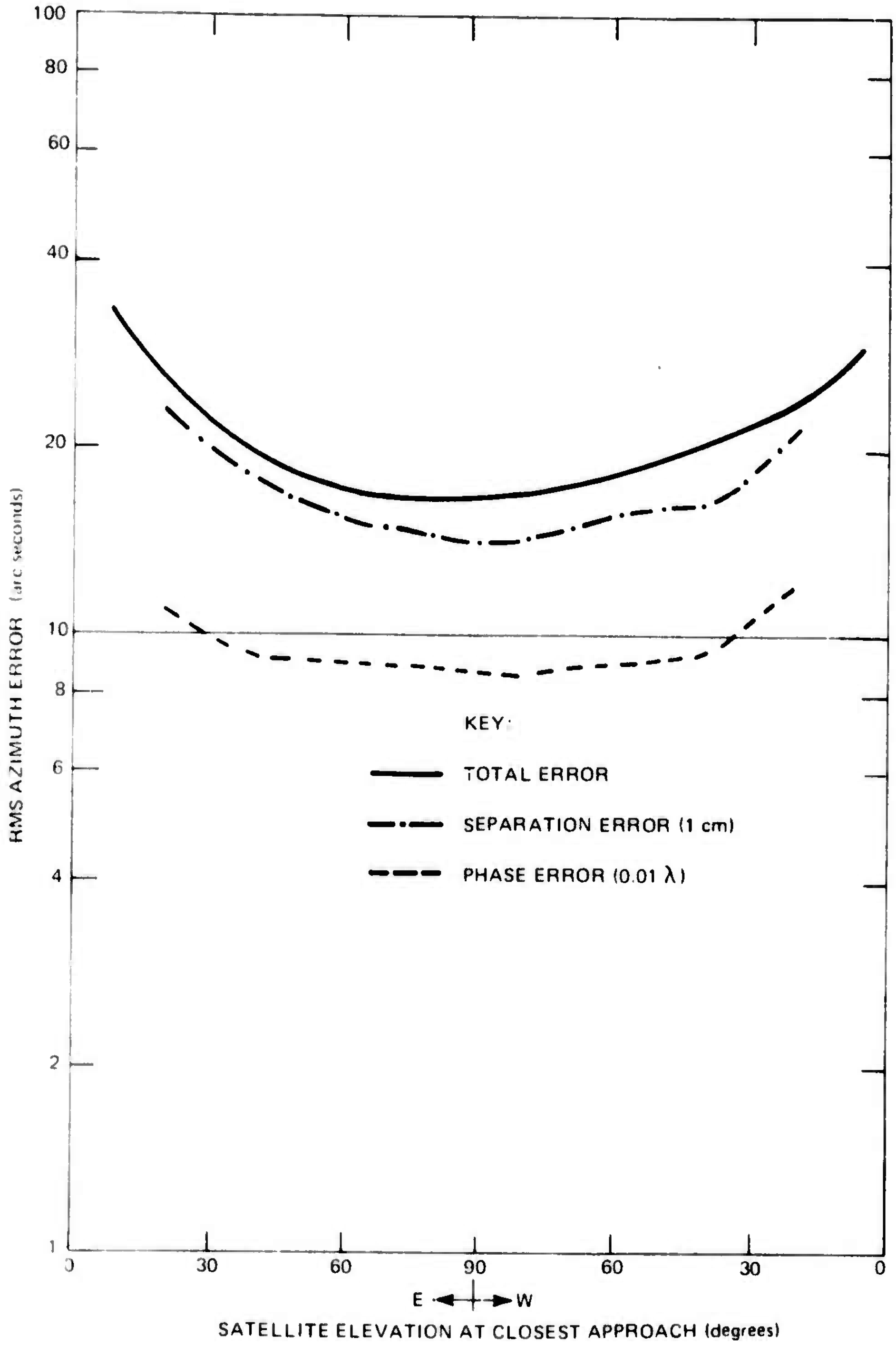

Fig. B-3 RMS AZIMUTH ERROR FOR SATELLITE PASSES OVER APL (AZ = 60°)

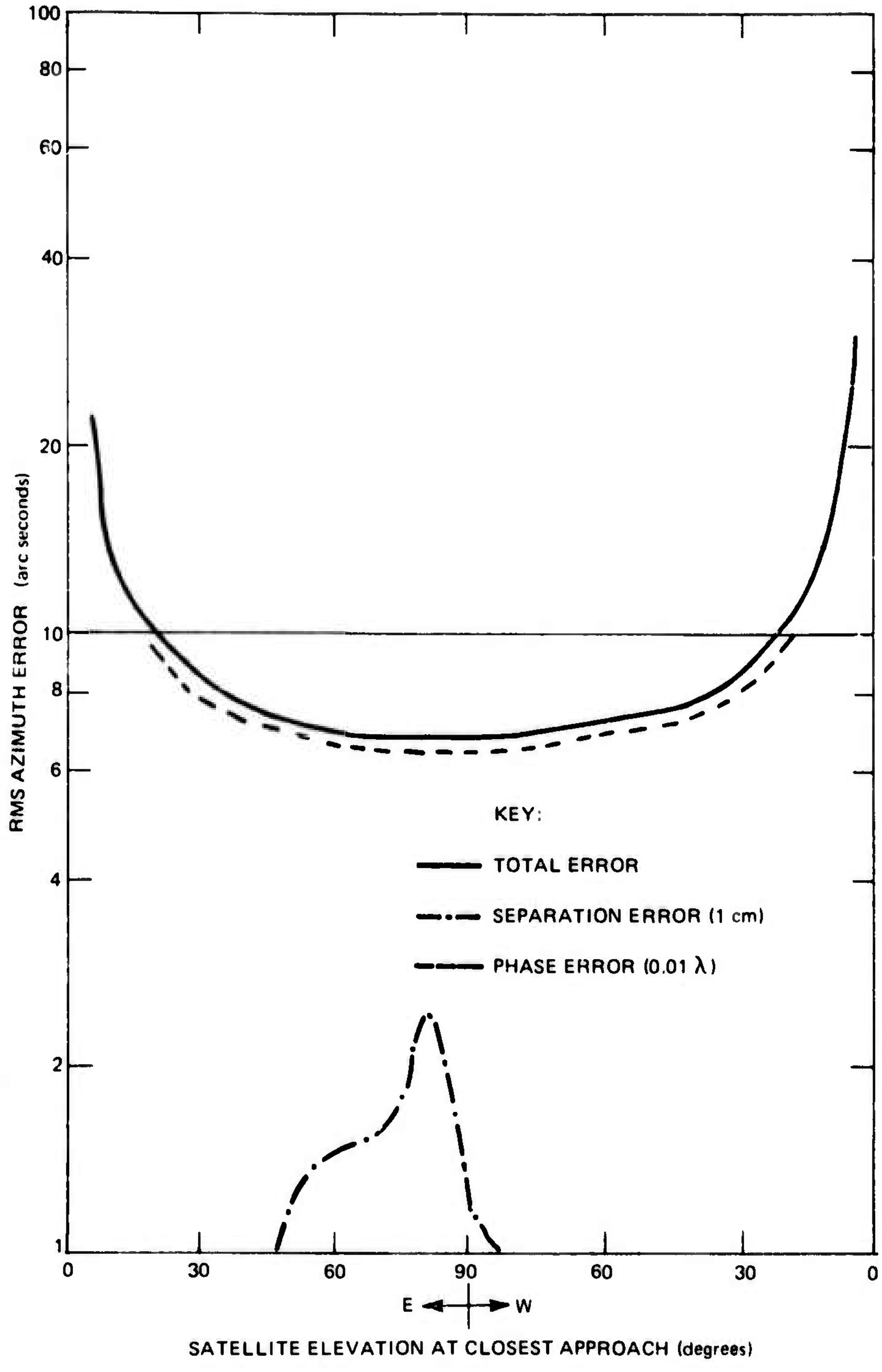

Fig. B-4 RMS AZIMUTH ERROR FOR SATELLITE PASSES OVER APL (AZ = 90°)

```
PL/I OPTIMIZING COMPILER        AZAL: PROCEDURE OPTIONS (MAIN);

                 SOURCE LISTING

 STMT LEV NT

    1      0  |AZAL: PROCEDURE OPTIONS (MAIN);
              |
    2   1  0  |      DCL XS(8),YS(8),ZS(8);
    3   1  0  |      DCL PX(8),PY(8),PZ(8);
    4   1  0  |      DCL PDX(8),PDY(8),PDZ(8);
    5   1  0  |      DCL ER(8),SAZ(8);
    6   1  0  |      DCL RR(8);
    7   1  0  |      DCL K(8,8) FLOAT;
    8   1  0  |      DCL KER(8,8) FLOAT;
    9   1  0  |      DCL RTK(8);
   10   1  0  |      DCL RTKER(8);
   11   1  0  |      DCL LAT FLOAT;
   12   1  0  |      DCL LON FLOAT;
   13   1  0  |      DCL H FLOAT;
   14   1  0  |      DCL MTR FLOAT;
   15   1  0  |      DCL N(2) CHAR(5) INIT('NORTH','SOUTH');
   16   1  0  |      DCL E(2) CHAR(4) INIT('EAST','WEST');
              |
              |                 /*   CONSTANTS      */
   17   1  0  |      RE=6378140D;
   18   1  0  |      WAVE=.74948234;       /*WAVELENGTH AT 400MHZ. */
   19   1  0  |       F=1/298.22;
   20   1  0  |      FF=(1-F)**2;
   21   1  0  |      DTR=ATAN(1)/ATAND(1);  /*DEG TO RADIANS */
   22   1  0  |      MTR=DTR/60;      /* MIN TO RADIANS */
   23   1  0  |      RTS=60/MTR;     /* RADIANS TO SECONDS*/
   24   1  0  |      DCL EL(0:9) INIT(-1,0,0,0,0,0,0,0,0,-1);
   25   1  0  |      PX=0;
   26   1  0  |      PY=0;
   27   1  0  |      PZ=0;
   28   1  0  |      PDX=0;
   29   1  0  |      PDY=0;
   30   1  0  |      PDZ=0;
              |
              |                 /*    INPUT DATA    */
   31   1  0  |      GET LIST(LAT,LON,HT);
   32   1  0  |      GET SKIP LIST(D,DH);
   33   1  0  |      GET SKIP LIST(ER,M);
   34   1  0  |      PUT SKIP EDIT('  ','LAT','LON','HT','XS','YS','ZS','D','DH','M')
              |          (X(11),A,X(6),2(A,X(8)),4(A,X(9)),A,X(10),A,X(9),A);
   35   1  0  |      PUT SKIP EDIT(ER,M) (X(19),2(F(7,3),X(4)),6(F(5,3),X(6)),F(8,2));
   36   1  0  |      PUT SKIP (4);
   37   1  0  |      PUT EDIT('LAT=',TRUNC(LAT),'DEG.',60*ABS(TRUNC(LAT)-LAT))
              |          (X(10),A,F(4),X(1),A,F(8,4),X(1),A);
   38   1  0  |      PUT SKIP(1);
   39   1  0  |      PUT EDIT('LON=',TRUNC(LON),'DEG.',60*ABS(TRUNC(LON)-LON))
```

Fig. B-5   LISTING OF AZTRAN ERROR ANALYSIS SIMULATION

```
PL/I OPTIMIZING COMPILER          AZAL: PROCEDURE OPTIONS (MAIN);

    STMT LEV NT

                        (X(10),A,F(4),X(1),A,F(8,4),X(1),A);
     40   1  0       PUT SKIP EDIT('HEIGHT=',HT,' M.')(X(10),A,F(7,2),X(1),A);
     41   1  0       PUT SKIP EDIT('D=',D,' M.')(X(10),A,F(6,2),A);
     42   1  0       PUT SKIP EDIT('DH=',DH,' M.')(X(10),A,F(6,2),A);
     43   1  0       PUT PAGE;
     44   1  0       ZN(1)=ER(1)/NE;
     45   1  0       ER(2)=ER(2)/(COSD(LAT)*NE);
     46   1  0       M=M*WAVE/360;

                                          /*    */
     47   1  0    RET:DO I=1 TO 8;
     48   1  1       GET SKIP LIST (XS(I),YS(I),ZS(I));
     49   1  1       END;
     50   1  0       CLA=COSD(LAT);
     51   1  0       CLC=COSD(LON);
     52   1  0       SLA=SIND(LAT);
     53   1  0       SLO=SIND(LON);
     54   1  0        N=RE/SQRT(CLA**2+FF*SLA**2);
     55   1  0       XN=(R+HT)*CLA*CLO;
     56   1  0       YN=(R+HT)*CLA*SLO;
     57   1  0       ZN=(R*FF+HT)*SLA;
     58   1  0       PX(1)=(R+HT)*SLA*CLO+R**3*F*(F-2)*CLA**2*SLA*CLO/RE**2;
     59   1  0       PX(2)=YN;
     60   1  0       PX(3)=-CLA*CLO;
     61   1  0       PX(4)=1;
     62   1  0       PY(1)=(N+HT)*SLA*SLO+R**3*F*(F-2)*CLA**2*SLA*SLO/RE**2;
     63   1  0       PY(2)=-XN;
     64   1  0       PY(3)=-CLA*SLC;
     65   1  0       PY(5)=1;
     66   1  0       PZ(1)=-(N*FF+HT)*CLA+R**3*FF*F*(F-2)*SLA**2*CLA/RE**2;
     67   1  0       PZ(3)=-SLA;
     68   1  0       PZ(6)=1;

     69   1  0       DO AZ=0 TO 90 BY 10;
     70   1  1       SAZ=0;
     71   1  1       K=0;
     72   1  1       RP=0;
     73   1  1       CA=COSD(AZ);
     74   1  1       SA=SIND(AZ);
     75   1  1       DX=-D*(SLA*CLO*CA+SLO*SA)+DH*CLA*CLO;
     76   1  1       DY=-D*(SLA*SLC*CA-CLO*SA)+DH*CLA*SLO;
     77   1  1       DZ=D*CIA*CA+DH*SLA;

                                /* COMPUTE PARTIALS  */
     78   1  1       PDXA=D*(SLA*CLO*SA-SLA*CA);
     79   1  1       PDYA=D*(SLA*SLC*SA+CLO*CA);
     80   1  1       PDZA=-D*CLA*SA;
     81   1  1       PDX(1)=-D*(CLA*CLO*CA)-DH*SLA*CLO;
     82   1  1       PDX(2)=-DY;
```

Fig. B-5 LISTING OF AZTRAN ERROR ANALYSIS SIMULATION (cont'd)

```
PL/I OPTIMIZING COMPILER        AZAL: PROCEDURE OPTIONS (MAIN);

 STMT LEV NT

   83   1  1 |      PDX(7)=-SLA*CLO*CA-SLO*SA;
   84   1  1 |      PDX(8)=CLA*CLO;
   85   1  1 |      PDY(1)=-D*CLA*SLO*CA-DH*SLA*SLO;
   86   1  1 |      PDY(2)=DX;
   87   1  1 |      PDY(7)=-SLA*SLO*CA+CLO*SA;
   88   1  1 |      PDY(8)=CLA*SLO;
   89   1  1 |      PDZ(1)=-D*SLA*CA+DH*CLA;
   90   1  1 |      PDZ(7)=CLA*CA;
   91   1  1 |      PDZ(8)=SLA;
   92   1  1 |      DO I=1 TO 8;
   93   1  2 |       X=XS(I)-XN;
   94   1  2 |       Y=YS(I)-YN;
   95   1  2 |       Z=ZS(I)-ZN;
   96   1  2 |      SR=SQRT(X**2+Y**2+Z**2);
             |
             |                /*  COMPUTE ELEVATION */
   97   1  2 |       A=X*XN+Y*YN+Z*ZN;
   98   1  2 |       B=SR*RE**2/R;
   99   1  2 |      EL(I)=ATAND(A*B/(B**2-A**2));
  100   1  2 |      IF EL(I)<0 THEN GO TO AA;
             |
             |                /*  COMPUTE AZIMUTH  */
  101   1  2 |       A=RE*(YS(I)*XN-XS(I)*YN);
  102   1  2 |       B=ZS(I)*RE**2-ZN*(XS(I)*XN+YS(I)*YN+ZS(I)*ZN);
  103   1  2 |      SAZ(I)=ATAND(A/B);
  104   1  2 |       A=ZS(I)/SQRT(XS(I)**2+YS(I)**2+ZS(I)**2);
  105   1  2 |       B=ABS(SLA);
  106   1  2 |      IF A<B THEN SAZ(I)=SAZ(I)+180;
  107   1  2 |      IF SAZ(I)<0 THEN SAZ(I)=SAZ(I)+360;
  108   1  2 |      RR(I)=-(X*PDXA+Y*PDYA+Z*PDZA)/SR;
  109   1  2 |      DO J=1 TO 8;
  110   1  3 |      A=(X*DX+Y*DY+Z*DZ)*(X*PX(J)+Y*PY(J)+Z*PZ(J))/SR**3;
  111   1  3 |      B=(X*PDX(J)+PX(J)*DX+Y*PDY(J)+PY(J)*DY+Z*PDZ(J)+PZ(J)*DZ)/SR;
  112   1  3 |      K(I,J)=A-B;
  113   1  3 |      END;       /* END J */
  114   1  2 | AA: END;       /* END I */
             |
             |              /*COMPUTE PASS LIMITS (G&H) */
  115   1  1 |      DO I=4 TO 0 BY -1;
  116   1  2 |      IF EL(I)<0 THEN GO TO BB;
  117   1  2 |      END;
  118   1  1 | BB: G=I+1;
  119   1  1 |      DO I=5 TO 9;
  120   1  2 |      IF EL(I)<0 THEN GO TO CC;
  121   1  2 |      END;
  122   1  1 | CC: H=I-1;
             |
             |              /*COMPUTE RR TRANSPOSE RR*/
  123   1  1 |      BTR=0;
```

Fig. B-5 LISTING OF AZTRAN ERROR ANALYSIS SIMULATION (cont'd)

```
PL/I OPTIMIZING COMPILER        AZAL: PROCEDURE OPTIONS (MAIN);

    STMT LEV NT

     124   1  1 |     DO I=G TO H;
     125   1  2 |     BTB=BTB+BB(I)**2;
     126   1  2 |     END;
                |
                |                /* COMPUTE BTK */
     127   1  1 |     BTK=0;
     128   1  1 |     DO J=1 TO 8;
     129   1  2 |     DO I=G TO H;
     130   1  3 |     BTK(J)=BTK(J)+BB(I)*K(I,J);
     131   1  3 |     END;
     132   1  2 |     END;
                |
                |              /* COMPUTE K EE EET KT */
     133   1  1 |     KEB=0;
     134   1  1 |     DO J=G TO H;
     135   1  2 |     DO I=G TO H;
     136   1  3 |     DO INDEX=1 TO 8;
     137   1  4 |     KEB(J,I)=KEB(J,I)+K(J,INDEX)*FB(INDEX)**2*K(I,INDEX);
     138   1  4 |     END;
     139   1  3 |     END;
     140   1  2 |     END;
                |
                |            /* ADD MEASUREMENT ERRORS */
     141   1  1 |     DO I=G TO H;
     142   1  2 |     KEB(I,I)=KEB(I,I)+M**2;
     143   1  2 |     END;
                |
                |       /* COMPUTE BTKEB */
     144   1  1 |     BTKEB=0;
     145   1  1 |     DO I=G TO H;
     146   1  2 |     DO J=G TO H;
     147   1  3 |     BTKEB(I)=BTKEB(I)+BB(J)*KEB(J,I);
     148   1  3 |     END;
     149   1  2 |     END;
                |
                |     /* COMPUTE DAZ */
     150   1  1 |     DAZ=0;
     151   1  1 |     DO I=G TO H;
     152   1  2 |     DAZ=DAZ+BTKEB(I)*BB(I);
     153   1  2 |     END;
     154   1  1 |     DAZ=RTS*SQRT(DAZ)/BTB;
     155   1  1 |     IF AZ>0 THEN GO TO DD;
     156   1  1 |     IF (SAZ(G)>90)&(SAZ(G)<270) THEN A1=2; ELSE A1=1;
     158   1  1 |     IF SAZ(G)<180 THEN A2=1; ELSE A2=2;
     160   1  1 |     IF (SAZ(H)>90)&(SAZ(H)<270) THEN A3=2; ELSE A3=1;
     162   1  1 |     IF SAZ(H)<180 THEN A4=1; ELSE A4=2;
     164   1  1 |     ELCA=0;
     165   1  1 |     DO I=G TO H;
     166   1  2 |     IF EL(I)>ELCA THEN ELCA=EL(I);
```

Fig. B-5 LISTING OF AZTRAN ERROR ANALYSIS SIMULATION (cont'd)

```
PL/I OPTIMIZING COMPILER         AZAL: PROCEDURE OPTIONS (MAIN);

    STMT LEV NT

     167   1  2         END;

     168   1  1         PUT SKIP(2);
     169   1  1        PUT EDIT(N(A1),E(A2),' TO ',N(A3),E(A4),' PASS ELEVATION AT CA IS'
                           ,ELCA,' DEG.')(X(10),6(A),F(5,1),A);
     170   1  1         PUT SKIP(2);
     171   1  1         PUT EDIT('ANT','ERROR IN SECONDS PER VARIABLE','RMS')
                           (X(10),A,X(9),A,X(66(,A(;
     172   1  1         PUT SKIP EDIT('AZ','LAT','LON','HT','XS','YS','ZS','D','DH','B',
                           'TOTAL'(
                           (X(11),A,X(6),2(A,X(8)),4(A,X(9)),A,X(10),A,X(9),A,X(8),A);
     173   1  1         PUT SKIP;
     174   1  1     DD: PUT SKIP EDIT(AZ,RTS*ABS(RTK*FR/RTR),RTS*R/SQRT(RTR),DAZ)
                           (X(11(,F(2),X(1),10(X(3),E(8,1))));
     175   1  1         END;        /* END    AZ*/
     176   1  0         PUT SKIP(4(;
     177   1  0         GO TO RET;
     178   1  0         END AZAL;
```

Fig. B-5 LISTING OF AZTRAN ERROR ANALYSIS SIMULATION (cont'd)